Calculating Yukawa Couplings in Heterotic Calabi-Yau Models

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Outline

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Introduction: Heterotic Yukawa couplings

- Consider heterotic string on CY 3-fold X

- observable bundle $V \to X$ with structure group $H \subset E_8$

- low-energy gauge group $G = \mathcal{C}_{E_8}(H)$

- matter multiplets from associated bundles $E_i \to X,\ i = 1, 2, 3$

Matter multiplets described by harmonic (0,1) forms:

$$\nu_i \in H^1(X, E_i) \qquad \bar{\partial}_{E_i} \nu_i = \bar{\partial}^{\dagger}_{E_i} \nu_i = 0 \qquad i = 1, 2, 3$$

Holomorphic Yukawa couplings:

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

Holomorphic Yukawa couplings are independent of representative:

$$\lambda(\nu_1 + \bar{\partial}_{E_1} a_1, \nu_2 + \bar{\partial}_{E_2} a_2, \nu_3 + \bar{\partial}_{E_3} a_3) = \lambda(\nu_1, \nu_2, \nu_3)$$

Holomorphic Yukawa couplings can be computed algebraically!

Normalisation (proportional to):

$$(\nu_i, \mu_i) := \int_X \nu_i \wedge \bar{\star}_{E_i} \mu_i = \frac{1}{2} \int_X J \wedge J \wedge \nu_i \wedge (H \bar{\mu}_i)$$

Normalization is not independent of representative and needs to be computed for harmonic (0,1) forms.

Algebraic computation (probably) not possible. Requires methods of differential geometry.

$$\lambda(\nu_1, \nu_2, \nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

- models from SO(10) GUTs: V has structure group $SU(4)$

Yukawa coupling $\mathbf{10}\,\mathbf{16}\,\mathbf{16}$:

$$\mathbf{10} \;\leftrightarrow\; \nu_1 \in H^1(X, \wedge^2 V)$$

$$\mathbf{16} \;\leftrightarrow\; \nu_{2,3} \in H^1(X, V)$$

$$\nu_1 \wedge \nu_2 \wedge \nu_3 \in H^3(X, \wedge^4 V) = H^3(X, \mathcal{O}_X) \cong \mathbb{C}$$

- models from SU(5) GUTs: V has structure group $SU(5)$

up-Yukawa $\mathbf{5}\,\mathbf{10}\,\mathbf{10}$:

$$\mathbf{5} \;\leftrightarrow\; \nu_1 \in H^1(X, \wedge^2 V^*)$$

$$\mathbf{10} \;\leftrightarrow\; \nu_{2,3} \in H^1(X, V)$$

$$\nu_1 \wedge \nu_2 \wedge \nu_3 \in H^3(X, \mathcal{O}_X) \cong \mathbb{C}$$

down-Yukawa $\mathbf{\bar{5}\,\bar{5}\,10}$:

$$\mathbf{\bar{5}} \;\leftrightarrow\; \nu_{1,2} \in H^1(X, \wedge^2 V)$$

$$\mathbf{10} \;\leftrightarrow\; \nu_3 \in H^1(X, V)$$

$$\nu_1 \wedge \nu_2 \wedge \nu_3 \in H^3(X, \wedge^5 V) = H^3(X, \mathcal{O}_X) \cong \mathbb{C}$$

SM Yukawa couplings are obtained after taking quotient by discrete symmetry Γ, adding a Wilson line and keeping the Γ-invariant parts.

We would like to....

- understand how to compute hol. Yukawa couplings using differential geometry language.
- clarify how such a differential geometry calculation relates to the algebraic one
- set the scene for a computation of the normalisation which requires differential geometry.

Yukawa couplings for co-dimension one CYs

Laboratory: tetra-quadric CY

defining polynomial p with multi-degree $\mathbf{q} = (2,2,2,2)$

$$X \sim \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right]$$

Line bundles $L = \mathcal{O}_X(k_1, \ldots k_4) = \mathcal{O}_{\mathcal{A}}(k_1, \ldots, k_4)|_X$

Consider line bundle sums

$$V = \bigoplus_{a=1}^{n} L_a \qquad n = 3, 4, 5 \qquad c_1(V) = 0$$

Leads to structure groups $S(U(1)^n) \subset SU(n)$ and gauge groups $E_6,\ SO(10),\ SU(5)\ \ \times S(U(1)^n)$

Tetra-quadric is simplest CICY which leads to line bundle standard models.

Line bundle cohomology on the tetra-quadric

Kozsul sequence: $0 \to N^* \otimes \mathcal{L} \to \mathcal{L} \to L \to 0$ where
$N = \mathcal{O}_{\mathcal{A}}(2,2,2,2)$ and $L = \mathcal{L}|_X$

$$\cdots \to H^1(\mathcal{A}, N^* \otimes \mathcal{L}) \xrightarrow{p} H^1(\mathcal{A}, \mathcal{L}) \xrightarrow{i^*} H^1(X, L) \ni \nu$$
$$\xrightarrow{\delta} H^2(\mathcal{A}, N^* \otimes \mathcal{L}) \xrightarrow{p} H^2(\mathcal{A}, \mathcal{L}) \to \cdots$$

$$\boxed{\nu = \hat{\nu}|_X\,, \quad \hat{\nu} \in H^1(\mathcal{A}, \mathcal{L})\,, \quad \bar{\partial}\hat{\nu} = 0}$$
"type 1"

$$H^1(X, L) \cong \mathrm{Coker}\left(H^1(\mathcal{A}, N^* \otimes \mathcal{L}) \xrightarrow{p} H^1(\mathcal{A}, \mathcal{L})\right) \oplus$$
$$\mathrm{Ker}\left(H^2(\mathcal{A}, N^* \otimes \mathcal{L}) \xrightarrow{p} H^2(\mathcal{A}, \mathcal{L})\right)$$

$$\boxed{\nu = \hat{\nu}|_X\,, \quad \hat{\omega} \in H^2(\mathcal{A}, N^* \otimes \mathcal{L})\,, \quad \bar{\partial}\hat{\nu} = p\hat{\omega}}$$
"type 2"

Holomorphic Yukawa couplings on the tetra-quadric

$$\lambda(\nu_1,\nu_2,\nu_3) = \int_X \Omega \wedge \nu_1 \wedge \nu_2 \wedge \nu_3$$

$$\nu_i \in H^1(X, K_i)$$
$$K_i = \mathcal{O}_X(\mathbf{k}_i)$$

$$= \frac{1}{\pi}\int_{\mathbb{C}^4} \frac{1}{p}\left(\bar\partial\hat\nu_1 \wedge \hat\nu_2 \wedge \hat\nu_3 - \hat\nu_1 \wedge \bar\partial\hat\nu_2 \wedge \hat\nu_3 + \hat\nu_1 \wedge \hat\nu_2 \wedge \bar\partial\hat\nu_3\right) \wedge dz_1 \wedge \cdots \wedge dz_4$$

insert $dp \wedge d\bar p \delta^2(p)$

use $\delta^2(p) d\bar p = \frac{1}{\pi}\bar\partial\left(\frac{1}{p}\right)$ and $\hat\Omega \wedge dp = dz_1 \wedge \cdots \wedge dz_4$

- Case 1: All ν_i are of type 1: $\Rightarrow \quad \bar\partial\hat\nu_i = 0$

$$\lambda(\nu_1,\nu_2,\nu_3) = 0$$

Yukawa couplings vanish due to structure of cohomology.

- Case 2: ν_3 is type 2, other ν_i are type 1:

$$\Rightarrow \quad \bar{\partial}\hat{\nu}_3 = p\hat{\omega}$$
$$\Rightarrow \quad \bar{\partial}\hat{\nu}_i = 0$$

$$\lambda(\nu_1, \nu_2, \nu_3) = \frac{1}{\pi}\int_{\mathbb{C}^4} \hat{\nu}_1 \wedge \hat{\nu}_2 \wedge \hat{\omega} \wedge dz_1 \wedge \cdots \wedge dz_4$$
$$= \frac{1}{\pi}\int_{\mathbb{C}^4} \kappa_1^{k_{1,1}} \kappa_2^{k_{2,2}} \kappa_3^{k_{3,3}-2} \kappa_4^{k_{3,4}-2} P_{(\mathbf{k}_1)} Q_{(\mathbf{k}_2)} R_{(\mathbf{k}_3-\mathbf{q})} d^4z\, d^4\bar{z}$$

Can always be explicitly integrated, or calculated algebraically:

$$\lambda(\nu_1, \nu_2, \nu_3) = 16\pi^3 c\mu(P, Q, R) \qquad\qquad \mu(P, Q, R) = \tilde{P}\tilde{Q}\tilde{R}$$

- Case 3: More than one ν_i originates from ambient 2-form

Slightly more complicated but can always be integrated.

Generalisation and a vanishing theorem

Co-dimension k CICY in ambient space $\mathcal{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$:

K defining polynomials $p_1, \ldots, p_k$, section of $\mathcal{N} = \bigoplus_{a=1}^{k} \mathcal{O}_{\mathcal{A}}(\mathbf{Q}_a)$

$$X \sim \left[\begin{array}{c|ccc} \mathbb{P}^{n_1} & Q_1^1 & \cdots & Q_k^1 \\ \vdots & \vdots & \vdots & \vdots \\ \mathbb{P}^{n_m} & Q_1^m & \cdots & Q_k^m \end{array} \right]$$

Line bundle cohomology for $K = \mathcal{K}|_X$, $\mathcal{K} \to \mathcal{A}$ from:

$$0 \to \wedge^k \mathcal{N}^* \otimes \mathcal{K} \to \wedge^{k-1} \mathcal{N}^* \otimes \mathcal{K} \to \cdots \to \mathcal{N}^* \otimes \mathcal{K} \to \mathcal{K} \to K \to 0$$

$\nu \in H^1(X, K)$ is called "type τ", where $\tau = 1, \ldots, k+1$, if it descends from $\hat{\omega} \in H^\tau(\mathcal{A}, \wedge^{\tau-1} \mathcal{N}^* \otimes \mathcal{K})$.

$$\lambda(\nu_1,\nu_2,\nu_3)=\int_X \Omega\wedge\nu_1\wedge\nu_2\wedge\nu_3$$ where ν_i are of type τ_i

Vanishing theorem:

If $\tau_1+\tau_2+\tau_3<\dim(\mathcal{A})$ then $\lambda(\nu_1,\nu_2,\nu_3)=0$.

Implies vanishing of many Yukawa couplings, in particular at higher co-dimension.

Formulation independent of embedding space?

Examples

Example 1: A tetra-quadric SO(10) model with vanishing Yukawa couplings

$$L_1 = \mathcal{O}_X(-1,0,0,1)\,, \quad L_2 = \mathcal{O}_X(0,-2,1,3)$$
$$L_3 = \mathcal{O}_X(0,0,1,-3)\,. \quad L_4 = \mathcal{O}_X(1,2,-2,-1)$$

spectrum: $8\ \mathbf{16}_2\,,\ 4\ \mathbf{16}_3\,,\ 3\ \mathbf{10}_{1,4}\,,\ 3\ \mathbf{10}_{2,3}$ plus singlets

Yukawa couplings: $\lambda_{IJK}\mathbf{10}_{1,4}^{(I)}\mathbf{16}_2^{(J)}\mathbf{16}_3^{(K)}$

$K_1 = L_1 \otimes L_4 = \mathcal{O}_X(0,2,-2,0)$ $\quad K_2 = L_2 = \mathcal{O}_X(0,-2,1,3)$ $\quad K_3 = L_3 = \mathcal{O}_X(0,0,1,-3)$

But all ν_i are of type 1 so that $\lambda_{IJK} = 0$.

Example 2: up-Yukawa couplings for SU(5) model on tetra-quadric

Standard model based on SU(5) GUT with line bundles

$$L_1 = \mathcal{O}_X(-1,0,0,1)\ ,\ L_2 = \mathcal{O}_X(-1,-3,2,2)\ ,\ L_3 = \mathcal{O}_X(0,1,-1,0)$$
$$L_4 = \mathcal{O}_X(1,1,-1,-1)\ ,\ L_5 = \mathcal{O}_X(1,1,0,-2)$$

spectrum: $8\,\mathbf{10}_2\ ,\ 4\,\mathbf{10}_5\ ,\ 4\,\overline{\mathbf{5}}_{2,4}\ ,\ 3\,\overline{\mathbf{5}}^H_{2,5}\ ,\ 8\,\overline{\mathbf{5}}_{4,5}\ ,\ 3\mathbf{5}^{\overline{H}}_{2,5}$

$12\,\mathbf{1}_{2,1}\ ,\ 12\,\mathbf{1}_{5,1}\ ,\ 20\,\mathbf{1}_{2,3}\ ,\ 12\,\mathbf{1}_{2,4}\ ,\ 4\,\mathbf{1}_{5,3}$

Relevant line bundles for up-Yukawa coupling:

$$\begin{array}{lll}
K_1 = L_2^* \otimes L_5^* & 3\ \mathbf{5}^H_{2,5} & \hat{\nu}_1 = \kappa_3^{-2} Q_{(0,2,-2,0)} d\bar{z}_3 \\
K_2 = L_5 & 4\ \mathbf{10}_2 & \hat{\nu}_2 = \kappa_4^{-2} R_{(1,1,0,-2)} d\bar{z}_4 \\
K_3 = L_2 & 8\ \mathbf{10}_5 & \hat{\omega} = \kappa_1^{-3}\kappa_2^{-5} S_{(-3,-5,0,0)} d\bar{z}_1 \wedge d\bar{z}_2
\end{array}$$

type 1

type 2

where

$$\begin{aligned}
Q &= q_0 + q_1 z_2 + q_2 z_2^2 \\
R &= r_0 + r_1 z_1 + r_2 z_2 + r_3 z_1 z_2 \\
S &= s_0 + s_1\bar{z}_2 + s_2\bar{z}_2^2 + s_3\bar{z}_2^3 + s_4\bar{z}_1 + s_5\bar{z}_1\bar{z}_2 + s_6\bar{z}_1\bar{z}_2^2 + s_7\bar{z}_1\bar{z}_2^3
\end{aligned}$$

Yukawa couplings, explicit calculation:

$$\begin{aligned}\lambda(Q,R,S) &= \frac{1}{\pi}\int_{\mathbb{C}^4}\frac{QRS}{\kappa_1^3\kappa_2^5\kappa_3^2\kappa_4^2}d^4z\,d^4\bar{z} \\ &= \frac{2\pi^3}{3}\,[3q_0r_0s_0+3q_0r_1s_4+q_0r_2s_1+q_0r_3s_5+q_1r_0s_1+q_1r_1s_5+ \\ &\quad q_1r_2s_2+q_1r_3s_6+q_2r_0s_2+q_2r_1s_6+3q_2r_2s_3+3q_2r_3s_7]\end{aligned}$$

Yukawa couplings, algebraic calculation:

$$\begin{aligned}\tilde{Q} &= q_0y_0^2+q_1y_0y_1+q_2y_1^2 \\ \tilde{R} &= r_0x_0y_0+r_1x_1y_0+r_2x_0y_1+r_3x_1y_1 \\ \tilde{S} &= s_0x_0y_0^3+s_1x_0y_0^2y_1+s_2x_0y_0y_1^2+s_3x_0y_1^3+s_4x_1y_0^3+s_5x_1y_0^2y_1+s_6x_1y_0y_1^2+s_7x_1y_1^3\end{aligned}$$

$$\begin{aligned}\mu(Q,R,S) &= \left(q_0\partial_{y_0}^2+q_1\partial_{y_0}\partial_{y_1}+q_2\partial_{y_1}^2\right)\left(r_0\partial_{x_0}\partial_{y_0}+r_1\partial_{x_1}\partial_{y_0}+r_2\partial_{x_0}\partial_{y_1}+r_3\partial_{x_1}\partial_{y_1}\right) \\ &\quad \left(s_0x_0y_0^3+s_1x_0y_0^2y_1+s_2x_0y_0y_1^2+s_3x_0y_1^3+s_4x_1y_0^3+s_5x_1y_0^2y_1+s_6x_1y_0y_1^2+s_7x_1y_1^3\right) \\ &= 2\,[3q_0r_0s_0+3q_0r_1s_4+q_0r_2s_1+q_0r_3s_5+q_1r_0s_1+q_1r_1s_5+ \\ &\quad q_1r_2s_2+q_1r_3s_6+q_2r_0s_2+q_2r_1s_6+3q_2r_2s_3+3q_2r_3s_7]\ .\end{aligned}$$

After taking quotient by $\Gamma = \mathbb{Z}_2 \times \mathbb{Z}_2$ and adding Wilson line:

$$\lambda^{(u)} = \frac{\pi^3}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Example 3: singlet-Yukawa couplings in SU(5) model on tetra-quadric

The same model has a coupling

$$\mathbf{1}_{2,4}\,\bar{\mathbf{5}}_{4,5}\,\mathbf{5}_{2,5}$$

with associated line bundles

$$\begin{array}{rcl}
K_1 = L_2 \otimes L_4^* = \mathcal{O}_X(-2,-4,3,3) & \to & 12\,\mathbf{1}_{2,4} \in \delta^{-1}\,\mathrm{Ker}\Big(H^2(\mathcal{O}_{\mathcal{A}}(-4,-6,1,1) \xrightarrow{p} H^2(\mathcal{O}_{\mathcal{A}}(-2,-4,3,3)\Big) \\
K_2 = L_4 \otimes L_5 = \mathcal{O}_X(2,2,-1,-3) & \to & 8\,\bar{\mathbf{5}}_{4,5} \in \delta^{-1}\,H^2(\mathcal{O}_{\mathcal{A}}(0,0,-3,-5) \\
K_3 = L_2^* \otimes L_5^* = \mathcal{O}_X(0,2,-2,0) & \to & 3\,\mathbf{5}_{2,5} \in H^1(\mathcal{O}_{\mathcal{A}}(0,2,-2,0))
\end{array}$$

type 2

type 1

and differential forms

$a_0, \ldots, a_{14}$

$$\begin{array}{rcl}
\hat{\omega}_1 & = & \kappa_1^{-4}\kappa_2^{-6} Q_{(-4,-6,1,1)} d\bar{z}_1 \wedge d\bar{z}_2 \quad \text{where} \quad \tilde{p}\tilde{Q} = 0 \\
\hat{\omega}_2 & = & \kappa_3^{-3}\kappa_4^{-5} R_{(0,0,-3,-5)} d\bar{z}_3 \wedge d\bar{z}_4 \\
\hat{\nu}_3 & = & \kappa_3^{-2} S_{(0,2,-2,0)} d\bar{z}_3\ .
\end{array}$$

b_0, b_1

Yukawa couplings for a 5-parameter family of tetra-quadrics:

$$\lambda(\nu_1,\nu_2,\nu_3) = -\frac{1}{\pi}\int_{\mathbb{C}^4} \frac{Q\mathcal{R}S}{\kappa_1^4\kappa_2^6\kappa_3^4\kappa_4^5} d^4z\, d^4\bar{z}$$

$$\begin{aligned} = \frac{\pi^3}{3240}\,&(2a_{14}b_1c_1 + 9a_{12}b_0c_2 + 9a_{13}b_0c_2 - 8a_4b_1c_2 - 8a_5b_1c_2 + 3a_{12}b_1c_2 + 3a_{13}b_1c_2 - 36a_7b_0c_3 - \\ &12a_2b_1c_3 - 12a_{14}b_0c_4 + 6a_2b_1c_4 + 6a_3b_1c_4 - 6a_6b_1c_4 - 6a_7b_1c_4 + 4a_{14}b_1c_4 - 36a_6b_0c_5 - \\ &12a_3b_1c_5 - 36a_2b_0c_6 - 36a_3b_0c_6 - 12a_6b_1c_6 - 12a_7b_1c_6)\end{aligned}$$

Still need to find kernel $M\mathbf{a} = 0$ where

$$M = \begin{pmatrix}
24c_6 & 0 & 0 & 0 & 4c_3 & 4c_6 & 0 & 0 & 0 & 24c_5 & 0 & 0 & 3c_4 & 0 & 0 \\
24c_5 & 0 & 6c_2 & 0 & 4c_6 & 4c_3 & 0 & 6c_2 & 0 & 24c_6 & 0 & 0 & -3c_4 & 0 & 0 \\
24c_4 & 24c_6 & 0 & 6c_2 & 4c_6-4c_4 & 4c_3+4c_4 & 6c_2 & 0 & 24c_5 & -24c_4 & 12c_2 & 0 & 3c_1 & 3c_4 & 2c_2 \\
0 & 24c_5 & 0 & 0 & 4c_3 & 4c_6 & 0 & 0 & 24c_6 & 0 & 12c_2 & 0 & 0 & -3c_4 & 2c_2 \\
24c_3 & 0 & 0 & 0 & 4c_6 & 4c_5 & 0 & 0 & 0 & 24c_6 & 0 & 12c_2 & -3c_4 & 0 & 2c_2 \\
24c_6 & 24c_4 & 6c_2 & 0 & 4c_4+4c_5 & 4c_6-4c_4 & 0 & 6c_2 & -24c_4 & 24c_3 & 0 & 12c_2 & 3c_4 & 3c_1 & 2c_2 \\
0 & 24c_3 & 0 & 6c_2 & 4c_5 & 4c_6 & 6c_2 & 0 & 24c_6 & 0 & 0 & 0 & 0 & -3c_4 & 0 \\
0 & 24c_6 & 0 & 0 & 4c_6 & 4c_5 & 0 & 0 & 24c_3 & 0 & 0 & 0 & 0 & 3c_4 & 0 \\
0 & 0 & 12c_6 & 12c_6 & 8c_2 & 8c_2 & 12c_3 & 12c_5 & 0 & 0 & 0 & 0 & 0 & 0 & 4c_4 \\
0 & 0 & 12c_5 & 12c_3 & 0 & 0 & 12c_6 & 12c_6 & 0 & 0 & 0 & 0 & 0 & 0 & -4c_4 \\
0 & 0 & 12c_6 & 12c_6 & 0 & 0 & 12c_5 & 12c_3 & 0 & 0 & 0 & 0 & 6c_2 & 6c_2 & 4c_4 \\
0 & 0 & 12c_3+12c_4 & 12c_4+12c_5 & 8c_2 & 8c_2 & 12c_6-12c_4 & 12c_6-12c_4 & 0 & 0 & 0 & 0 & 6c_2 & 6c_2 & 4c_1-4c_4
\end{pmatrix}$$

The Yukawa coupling

$$\lambda_{ij} S^i L^j \bar{H}$$

then becomes

$$\lambda = \frac{\pi^3}{180} \begin{pmatrix} 0 & (c_3 - c_5)\left(4c_4^2 + c_1\left(c_3 + c_5 - 2c_6\right)\right)(c_3 + c_5 + 2c_6) \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

This is generically rank 1, but will be generally rank 2 away from the 5-parameter family. For $c_3 = c_5$ the Higgs remains massless even if $\langle S^i \rangle \neq 0$.

Yukawa unification?

Consider, for example, $SU(5)$ GUT with $\Gamma = \mathbb{Z}_2$ and down-Yukawa:

upstairs: 6 families

downstairs: 3families

$$\sum_{I,J=1}^{6} \lambda_{IJ} \bar{\mathbf{5}}^H \, \bar{\mathbf{5}}^I \, \mathbf{10}^J \quad \to \quad \sum_{i,j=1}^{3} \lambda_{ij}^{(d)} H d^i Q^j$$

$$\to \quad \sum_{i,j=1}^{3} \lambda_{ij}^{(e)} H L^i e^j$$

Wilson line described by Γ-representations χ_2, χ_3 satisfying $\chi_2^2 \otimes \chi_3^3 = 1$. For $\Gamma = \mathbb{Z}_2$ we have $\chi_2 = (1)$ and $\chi_3 = (0)$.

$$\chi_H = \chi_2^* = (1) \qquad \chi_d = \chi_3^* = (0) \qquad \chi_Q = \chi_2 \otimes \chi_3 = (1)$$

$$\chi_L = \chi_2^* = (1) \qquad \chi_e = \chi_2 \otimes \chi_2 = (0)$$

$$\begin{array}{cc} & \begin{array}{ccc} \chi_{\mathbf{10}} & (1) & (0) \end{array} & \chi_{\bar{\mathbf{5}}} \\ (\lambda_{IJ}) = & \begin{pmatrix} \lambda_{(0,1)} & \lambda_{(0,0)} \\ \lambda_{(1,1)} & \lambda_{(1,0)} \end{pmatrix} & \begin{array}{c} (0) \\ (1) \end{array} \end{array}$$

$\lambda^{(d)}$ $\qquad\qquad$ $\lambda^{(e)}$

$\lambda^{(e)}$ and $\lambda^{(d)}$ are (in general) unrelated!

This holds for any symmetry Γ and all types of Yukawa couplings.

In heterotic GUT models with Wilson line breaking Yukawa unification in the traditional sense (i.e. enforced by the GUT symmetry) never arises.

Q: Can Yukawa unification arise from additional symmetries of the upstairs theory, e.g. from Γ and additional U(1)'s?

A unification scenario for an SU(5) line bundle model

Line bundle sum $V = \bigoplus_{a=1}^{5} L_a$

Upstairs gauge group $SU(5) \times \hat{J},\ \hat{J} = S(U(1)^5)$ and $\Gamma = \mathbb{Z}_2$

Assumed spectrum: $\mathcal{V}_{\mathbf{10}} = \mathrm{Span}(\mathbf{10}_4, \mathbf{10}_5)\ ,\quad \mathcal{V}_{\bar{\mathbf{5}}} = \mathrm{Span}(\bar{\mathbf{5}}^H_{1,2}, \bar{\mathbf{5}}_{3,4}, \bar{\mathbf{5}}_{3,5})$

$\hat{J}$-representations:
$$\begin{aligned} R_{\mathbf{10}}(\boldsymbol{\alpha}) &= \mathrm{diag}\left(e^{i\mathbf{e}_4\cdot\boldsymbol{\alpha}}, e^{i\mathbf{e}_5\cdot\boldsymbol{\alpha}}\right) \\ R_{\bar{\mathbf{5}}}(\boldsymbol{\alpha}) &= \mathrm{diag}\left(e^{i(\mathbf{e}_1+\mathbf{e}_2)\cdot\boldsymbol{\alpha}}, e^{i(\mathbf{e}_3+\mathbf{e}_4)\cdot\boldsymbol{\alpha}}, e^{i(\mathbf{e}_3+\mathbf{e}_5)\cdot\boldsymbol{\alpha}}\right) \end{aligned}$$

$\mathbb{Z}_2$-representations: $\rho_{\mathbf{10}}(-1) = \sigma\ ,\qquad \rho_{\bar{\mathbf{5}}}(-1) = \mathrm{diag}(-1, \sigma)\ ,\qquad \sigma = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Upstairs Yukawa coupling: $\hat{W} = \bar{\mathbf{5}}^H_{1,2}\,(\bar{\mathbf{5}}_{3,4}, \bar{\mathbf{5}}_{3,5})\,\hat{Y} \begin{pmatrix} \mathbf{10}_4 \\ \mathbf{10}_5 \end{pmatrix}$

$$\hat{Y} = 2Y = \begin{pmatrix} 0 & y \\ y' & 0 \end{pmatrix}$$

Downstairs this leads to Yukawa unification, $Y^{(e)} = Y^{(d)}$, for one family.

I this way, Yukawa unification can be engineered for one family but not the other two.

An example for Yukawa unification

Manifold:

$$\hat{X} \sim \left[\begin{array}{c|ccccc} \mathbb{P}^1 & 1 & 0 & 1 & 0 & 0 \\ \mathbb{P}^1 & 1 & 0 & 1 & 0 & 0 \\ \mathbb{P}^1 & 0 & 1 & 0 & 0 & 1 \\ \mathbb{P}^1 & 0 & 1 & 0 & 0 & 1 \\ \mathbb{P}^2 & 1 & 0 & 0 & 1 & 1 \\ \mathbb{P}^2 & 0 & 1 & 1 & 1 & 0 \end{array}\right]^{6,26}_{-40} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \longrightarrow x_{i,\alpha} \\ \longrightarrow \mathbf{y} = (y_0, y_1, y_2)^T \\ \longrightarrow \mathbf{z} = (z_0, z_1, z_2)^T \end{array}$$

$$\mathcal{N}_1 \cdots\cdots \mathcal{N}_5$$

$\mathbb{Z}_2$-symmetry: $x_{i,\alpha} \to (-1)^{\alpha+1} x_{i,\alpha}\ , \quad \mathbf{y} \leftrightarrow \mathbf{z}$

$$\mathcal{N}_1 \leftrightarrow \mathcal{N}_3\ , \quad \mathcal{N}_2 \leftrightarrow \mathcal{N}_5\ , \quad \mathcal{N}_4 \to \mathcal{N}_4$$

line bundles: $L_1 = \mathcal{O}_{\hat{X}}(-1,0,-1,1,0,0)\ , \quad L_2 = \mathcal{O}_{\hat{X}}(2,1,2,0,-1,-1)\ , \quad L_3 = \mathcal{O}_{\hat{X}}(1,1,-1,-1,0,0)\ ,$
$L_4 = \mathcal{O}_{\hat{X}}(-1,-1,0,0,0,1)\ , \quad L_5 = \mathcal{O}_{\hat{X}}(-1,-1,0,0,1,0)\ .$

spectrum: $\mathbf{10}_4\ , \quad \mathbf{10}_5\ , \quad 2\,\bar{\mathbf{5}}^H_{1,2}\ , \quad \bar{\mathbf{5}}_{3,4}\ , \quad \bar{\mathbf{5}}_{3,5}$
$6\,\mathbf{10}_2\ , \quad 2\,\bar{\mathbf{5}}_{1,3}\ , \quad 2\,\mathbf{5}_{1,4}\ , \quad 2\,\mathbf{5}_{1,5}\ , \quad 7\,\bar{\mathbf{5}}_{4,5}\ , \quad \mathbf{5}_{4,5}$

Right multiplets and representations of $S(U(1)^5)$ and $\mathbb{Z}_2$ for unification scenario:

$$\hat{W} = \bar{\mathbf{5}}^H_{1,2}\,(\bar{\mathbf{5}}_{3,4}, \bar{\mathbf{5}}_{3,5})\,\hat{Y}\begin{pmatrix} \mathbf{10}_4 \\ \mathbf{10}_5 \end{pmatrix}\,, \qquad \hat{Y} = 2Y = \begin{pmatrix} 0 & y \\ y' & 0 \end{pmatrix}$$

type 4 type 2

Vanishing theorem does not apply!

We have checked by explicit computation, using the above methods, that $y, y' \neq 0$.

Conclusion

- We can compute the holomorphic (perturbative) Yukawa couplings for heterotic line bundle models on CICYs, both algebraically and in terms of differential geometry.

- Complex structure dependence can be worked out explicitly.

- Many Yukawa couplings are zero perturbatively due to a topological vanishing theorem.

- Underlying GUT symmetry of models never leads to Yukawa unification.

- Partial Yukawa unification can be induced by an interplay of discrete and U(1) symmetries.

- Generalisation to CYs defined in more general toric ambient spaces and to bundles with non-Abelian structure group possible.

- Most pressing outstanding problem is the calculation of the matter field Kahler metric -> physical Yukawa couplings.

Thanks