$(0,2)$ SCFTs from Wrapped Branes

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Motivation

- Large classes of interacting p-dimensional superconformal field theories (SCFTs) obtained from a d-dimensional theory "compactified" on a manifold $\mathcal{M}_q$ of dimensions $q = d - p$. [Vafa-Witten], [Witten], [Bershadsky-Johansen-Sadov-Vafa], [Klemm-Lerche-Mayr-Vafa-Warner], [Maldacena-Núñez], [Kapustin], [Gaiotto], [Gaiotto-Moore-Neitzke], [Dimofte-Gukov-Gaiotto], [Cecotti-Cordova-Vafa], ...

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- Strings and branes provide a useful unifying picture. The p-dimensional theory is the low-energy limit of M/D-branes wrapped on $\mathcal{M}_q$. Some supersymmetry is preserved by a partial topological twist - naturally incorporated by the brane construction. [Bershadsky-Sadov-Vafa]

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- This setup leads to interesting "dualities" between the p-dimensional SCFT and a (topological) theory on $\mathcal{M}_q$. [Alday-Gaiotto-Tachikawa], [Gadde-Pomoni-Rastelli-Razamat], [Dimofte-Gaiotto-Gukov], [Cecotti-Cordova-Vafa], ...

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- The p-dimensional SCFTs typically admit a "large N" limit and have holographic duals which can be explicitly constructed. New examples of AdS/CFT.

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- Evidence for many new 2d $(0, 2)$ SCFTs from D3-branes on a Riemann surface and M5-branes on a four-manifold.
- c-extremization - a general principle in 2d $(0, 2)$ SCFTs which determines the exact superconformal R-symmetry. Similar to a-maximization in 4d and F-maximization in 3d. [Intriligator-Wecht], [Jafferis], [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg]

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Disclaimer: Here I always take $\mathcal{M}_q$ to be compact of constant curvature for $q = 2$ and compact and Einstein for $q = 4$. Generalizations are possible and very interesting! [Anderson-Beem-NB-Rastelli], [Gaiotto-Maldacena]

Basic idea

Supersymmetry is generically broken when a QFT is placed on a curved manifold. To preserve (some) supersymmetry perform a “topological twist”, i.e. use the R-symmetry to cancel the space-time curvature [Witten]

$$A_\mu^R = -\frac{1}{4}\omega_\mu \ , \qquad \rightarrow \qquad \widetilde{\nabla}_\mu \epsilon = \left(\partial_\mu + \frac{1}{4}\omega_\mu + A_\mu^R\right)\epsilon = \partial_\mu \epsilon = 0 \ .$$

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Related recent work: [Kutasov-Lin], [Gadde-Gukov-Putrov], [Franco-Ghim-Lee-Seong-Yokoyama], [Maxfield-Robbins-Sethi], [Putrov-Song-Yan], [Núñez et al], [Baggio-Halmagyi-Mayerson-Robbins-Wecht], [Assel-Schäfer-Nameki-Wong], [Gadde-Razamat-Willet], ...

Plan

- Introduction ✓
- c-extremization
- D3-branes on $\Sigma_{\mathfrak{g}}$
- M5-branes on $\mathcal{M}_4$
- Outlook

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$$\nabla^\mu J^I_\mu = \sum_{L=1}^{n} \frac{k^{IL}}{8\pi} F^L_{\mu\nu} \varepsilon^{\mu\nu} \,, \qquad \nabla_\mu T^{\mu\nu} = \frac{k}{96\pi} g^{\nu\alpha} \varepsilon^{\sigma\rho} \partial_\sigma \partial_\beta \Gamma^\beta_{\alpha\rho} \,.$$

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The symmetric tensor k^{IL} and the constant k are the 't Hooft anomaly coefficients

$$k^{IL} = \mathrm{Tr}_{\text{“Weyl”}}\gamma^3 Q^I Q^L \,, \qquad k = \mathrm{Tr}_{\text{“Weyl”}}\gamma^3 \,.$$

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Important assumptions: The CFT is unitary with a normalizable vacuum.

c-extremization

Consider a trial R-current Ω_μ^{tr}

$$\Omega_\mu^{\text{tr}}(t) = \Omega_\mu + \sum_{I\,(\neq R)} t_I J_\mu^I \,.$$

Then construct a trial central charge $c_r^{\text{tr}}(t)$ proportional to the anomaly of the trial R-symmetry:

$$c_r^{\text{tr}}(t) = 3\left(k^{RR} + 2\sum_{I\neq R} t^I k^{RI} + \sum_{I,L\neq R} t^I t^L k^{IL}\right) \,.$$

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This is an extremality condition for $c_R^{\text{tr}}(t)$.

$$\frac{\partial c_r^{\text{tr}}(t^*)}{\partial t^I} = 0 \ , \quad \forall\, I \neq R \ , \quad \rightarrow \quad c_r^{\text{tr}}(t^*) = c_r \ .$$

Similar to a-maximization in 4d and F-maximization in 3d. [Intriligator-Wecht], [Jafferis], [Closset-Dumitrescu-Festuccia-Komargodski-Seiberg]

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Caveat: May fail if there are accidental global symmetries in the IR.

$$4 = 2 + 2$$

D3-branes on a Riemann surface

Study $\mathcal{N} = 4$ SYM with gauge group G on $\mathbb{R}^{1,1} \times \Sigma_{\mathfrak{g}}$ with a partial topological twist on $\Sigma_{\mathfrak{g}}$. [Bershadsky-Johansen-Sadov-Vafa], [Maldacena-Núñez]

Turn on an $SO(2)^3 \in SO(6)_R$ background gauge field

$$F = -T\text{vol}_{\Sigma_{\mathfrak{g}}} \,, \qquad T = a_1 T_1 + a_2 T_2 + a_3 T_3 \;.$$

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This construction is realized in string theory (for $G = SU(N)$) by N D3-branes wrapping a holomorphic 2-cycle in a CY_4 which is a sum of complex line bundles with degrees ℓ_i over $\Sigma_{\mathfrak{g}}$

$$\mathcal{L}_1 \oplus \mathcal{L}_2 \oplus \mathcal{L}_3 \to \text{CY}_4 \to \Sigma_g \ .$$

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Use anomalies and c-extremization to argue that the 2d theory is a $(0,2)$ CFT for a wide range of parameters.

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The 2D theory inherits the $SO(2)^3$ symmetry. The trial R-symmetry is

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The number of 2D massless fermions is (index theorem)

$$n_R^{(\sigma)} - n_L^{(\sigma)} = \frac{1}{2\pi} \int_{\Sigma_\mathfrak{g}} \mathrm{Tr}_\sigma F \ , \qquad c_r^{tr}(\epsilon) = 3 d_G \sum_\sigma (n_R^{(\sigma)} - n_L^{(\sigma)})(q_R^{(\sigma)}(\epsilon))^2 \ .$$

Here $\sigma \in$ **4** of $SU(4)_R$ and labels the 4d fermions of $\mathcal{N} = 4$ SYM.

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Extremize with respect to ϵ_i to find

$$c_r = 12 \eta_\Sigma d_G \frac{a_1 a_2 a_3}{(a_1 + a_2 + a_3)^2 - 2(a_1^2 + a_2^2 + a_3^2)} \ , \qquad \eta_\Sigma = \begin{cases} 2|\mathfrak{g} - 1| & \mathfrak{g} \neq 1 \\ 1 & \mathfrak{g} = 1 \end{cases}$$

Note that $c_r - c_\ell = 0$, i.e. no gravitational anomaly.

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$$n_R^{(\sigma)} - n_L^{(\sigma)} = \frac{1}{2\pi} \int_{\Sigma_{\mathfrak{g}}} \mathrm{Tr}_\sigma F \ , \qquad c_r^{tr}(\epsilon) = 3 d_G \sum_\sigma (n_R^{(\sigma)} - n_L^{(\sigma)}) (q_R^{(\sigma)}(\epsilon))^2 \ .$$

Here $\sigma \in \mathbf{4}$ of $SU(4)_R$ and labels the 4d fermions of $\mathcal{N} = 4$ SYM.

Extremize with respect to ϵ_i to find

$$c_r = 12 \eta_\Sigma d_G \frac{a_1 a_2 a_3}{(a_1 + a_2 + a_3)^2 - 2(a_1^2 + a_2^2 + a_3^2)} \ , \qquad \eta_\Sigma = \begin{cases} 2|\mathfrak{g} - 1| & \mathfrak{g} \neq 1 \\ 1 & \mathfrak{g} = 1 \end{cases}$$

Note that $c_r - c_\ell = 0$, i.e. no gravitational anomaly.

c-extremization is essential to finding the correct answer. Without it one finds a mismatch between the supergravity and field theory result for the central charges. [Almuhairi-Polchinski]

Supergravity solutions - I

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The IIB metric is (there is only 5-form flux)

$$ds^2_{10} = \Delta^{1/2}\left[e^{2f}\left(\frac{-dt^2+dz^2+dr^2}{r^2}\right)+e^{2g}\,ds^2_{\Sigma_{\mathfrak{g}}}\right]$$

$$+\,\Delta^{-1/2}\sum_{i=1}^{3}(X_i)^{-1}\left(d\mu_i^2+\mu_i^2\left(d\varphi_i+A^{(i)}\right)^2\right)\,,$$

where

$$\Delta=\sum_{i=1}^{3}X_i\mu_i^2\,,\qquad X_1X_2X_3=1\,,\qquad \sum_{i=1}^{3}\mu_i^2=1\,,$$

The parameters specifying the solution are

$$e^{2g} = \frac{a_1X_2+a_2X_1}{2}\,,\qquad (X_1)^2X_2=\frac{a_1(a_2+a_3-a_1)}{a_3(a_1+a_2-a_3)}\,,$$

$$e^{f} = \frac{2}{X_1+X_2+X_3}\,,\qquad X_1(X_2)^2=\frac{a_2(a_1+a_3-a_2)}{a_3(a_1+a_2-a_3)}\,.$$

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Holographic RG flows from an asymptotically locally $AdS_5\times S^5$ space to these AdS_3 fixed points.

Supergravity solutions - II

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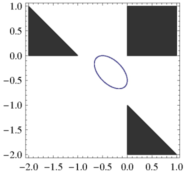
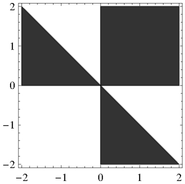
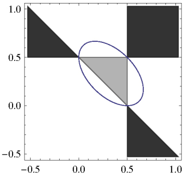




$$\underline{\mathfrak{g} = 0, 1 :} \qquad \{a_1 > 0,\ a_2 > 0\} \cup \{a_1 > 0,\ a_3 > 0\} \cup \{a_2 > 0,\ a_3 > 0\}\ .$$



$$\underline{\mathfrak{g} > 1 :} \qquad \{a_1 > \tfrac{1}{2},\ a_2 > \tfrac{1}{2}\} \cup \{a_1 > \tfrac{1}{2},\ a_3 > \tfrac{1}{2}\} \cup \{a_2 > \tfrac{1}{2},\ a_3 > \tfrac{1}{2}\}$$
$$\cup \{a_1 < \tfrac{1}{2},\ a_2 < \tfrac{1}{2},\ a_3 < \tfrac{1}{2}\} \cup \{a_1 = a_2 = \tfrac{1}{2},\ a_3 = 0 + \text{permutations}\}\ .$$

Salient features

- For $a_1 = a_2 = 0$ and $a_3 = 1$ supersymmetry is enhanced to $(4,4)$. The SCFT is a σ-model on the Hitchin moduli space on $\Sigma_{\mathfrak{g}}$. [Bershadsky-Johansen-Sadov-Vafa]

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- Special twist for $a_1 = a_2 = a_3 = \frac{1}{3}$ with $(0,2)$ supersymmetry and $SU(3)_F \times U(1)_R$ global symmetry

  $$c_r = c_\ell = \frac{8}{3}(\mathfrak{g}-1)d_G = \frac{32}{3}(\mathfrak{g}-1)a_{\mathcal{N}=4} \ .$$

Salient features

- The central charges computed in supergravity match with the ones in field theory.

$$c_r = c_\ell = 24(\mathfrak{g} - 1)N^2 \, \frac{a_1\, a_2\, a_3}{(a_1 + a_2 + a_3)^2 - 2(a_1^2 + a_2^2 + a_3^2)} \; .$$

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- Not clear what is the IR dynamics of the 2d $(0,2)$ theories with values of a_i for which there are no AdS_3 vacua.

4d $\mathcal{N} = 1$ SCFTs on $\Sigma_{\mathfrak{g}}$

Large classes of 4d $\mathcal{N} = 1$ SCFTs can be obtained by placing D3-branes on CY_3 singularities in IIB string theory. [Klebanov-Witten], [Morrison-Ronen Plesser], [Acharya-Figueroa-O'Farrill-Hull-Spence]. . .

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Obtain 2d $(0,2)$ SCFTs by placing these theories on a Riemann surface with a partial topological twist and background flavor symmetry fluxes.

These SCFTs have $SU(2) \times U(1)_F \times U(1)_B \times U(1)_R$ global symmetry, so turn on a background magnetic field along $\Sigma_\mathfrak{g}$

$$T = b_1 T_1 + b_2 T_2 + \mathfrak{b} T_B + \frac{\kappa}{2} T_R \ .$$

For every 4d $\mathcal{N} = 1$ SCFTs there is a 4-parameter family of 2d theories $\{b_1, b_2, \mathfrak{b}, \mathfrak{g}\}$.

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For the $SU(N)$ $Y^{p,0}$ theory one finds

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A new feature: The $U(1)_B$ mixes with the R-symmetry along the RG flow to 2d.

We have constructed $AdS_3 \times_w \mathcal{M}_7$ solutions of IIB supergravity dual to (some of) these 2d $(0,2)$ SCFTs. The internal manifolds $\mathcal{M}_7$ have interesting geometry.

An universal RG flow across dimensions.

For $b_1 = b_2 = \mathfrak{b} = 0$ we find

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The IR 2d superconformal R-symmetry coincides with the UV 4d one, and no mixing with $U(1)_F$ flavor symmetries occurs along the RG flow (if $k_F = 0$).

For such an RG flow across dimensions we obtain a universal relation

$$\begin{pmatrix} c_r \\ c_l \end{pmatrix} = \frac{16}{3}(\mathfrak{g}-1)\begin{pmatrix} 5 & -3 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} a \\ c \end{pmatrix}\ .$$

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Note that for theories with $a = c$, one has $c_r = c_l$ and

$$c_r = \frac{32}{3}(\mathfrak{g}-1)\, a\ .$$

This universal relation is also reproduced holographically.

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An intricate web of holographic RG flows connecting $\mathcal{N}=4$ SYM, the 4d $\mathcal{N}=1$ LS CFT and the family of 2d $(0,2)$ SCFTs. This web is parametrized by the mass $\mathfrak{m}$ and the “flavor flux” $\mathfrak{b}$.

$$6 = 2 + 4$$

M5-branes on four-manifolds

Study M5-branes (i.e. 6d (2,0) theory) wrapping calibrated 4-cycles, $\mathcal{M}_4$, in special holonomy manifolds (2d theories of class $\mathcal{Q}$ ⌣̈).

- $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ in $CY_4 \rightarrow$ (at least) $(0,2)$ SCFTs. Class $\mathcal{S}$ SCFTs on a Riemann surface. Dual supergravity solutions exist when either $\mathfrak{g}_1 > 1$ or $\mathfrak{g}_2 > 1$.
- Kähler 4-cycle in $CY_4 \rightarrow (0,2)$ SCFTs. [Ganor] Dual supergravity solutions exist for negatively curved Kähler-Einstein metric on $\mathcal{M}_4$. One parameter family of SCFTs.
- Kähler SLAG 4-cycle in $HK_2 \rightarrow (1,2)$ SCFTs. Dual supergravity solution exists for $\mathcal{M}_4 = \mathbb{CH}^2/\Gamma$. [Gauntlett-Kim]
- Co-associative 4-cycle in $G_2 \rightarrow (0,2)$ SCFTs. Dual supergravity solution exists for $\mathcal{M}_4$ Einstein with ASD Weyl tensor. [Gauntlett-Kim-Waldram]

These are all twists with at least $(0,2)$ supersymmetry. Other constructions with less supersymmetry are possible. [Gauntlett-Kim-Waldram]

M5-branes on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$

The $(2,0)$ theory has $SO(5)$ R-symmetry with Cartan generators $T_{A,B}$. Turn on background flux

$$T_s = \frac{-\kappa_s + z_s}{2} T_A + \frac{-\kappa_s - z_s}{2} T_B \ , \qquad\qquad s = 1, 2 \ ,$$

along $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$ with $(\mathfrak{g}_s - 1) z_s \in \mathbb{Z}$.

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Use the anomaly polynomial of the $(2,0)$ theory of type G [Witten], [Harvey-Minasian-Moore], [Intriligator], [Yi]

$$I_8 = \frac{r_G}{48} \left[p_2(\mathfrak{N}) - p_2(\mathfrak{T}) + \frac{1}{2} \left(p_1(\mathfrak{N}) - p_1(\mathfrak{T}) \right)^2 \right] + \frac{d_G h_G}{24} p_2(\mathfrak{N}) \,,$$

Integrate the M5-brane anomaly polynomial I_8 over $\mathcal{M}_4$ to find the anomaly polynomial in 2d [Alday-Benini-Tachikawa]

$$I_4 = \frac{c_r}{6} c_1(F_R) \wedge c_1(F_R) - \frac{c_r - c_\ell}{24} p_1(\mathfrak{T}_2) \,,$$

to calculate the central charges of these SCFTs. Again, c-extremization is essential.

M5-branes on $\Sigma_{\mathfrak{g}_1} \times \Sigma_{\mathfrak{g}_2}$

For the A_N $(2,0)$ theory with $N \gg 1$ and $\mathfrak{g}_{1,2} > 1$ one finds

$$c_r \approx c_\ell \approx 2(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)\frac{3z_1^2 z_2^2 + z_1^2 + z_2^2 - 8z_1 z_2 + 3}{1 - 3z_1 z_2} N^3 \, .$$

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- One can find AdS_3 supergravity duals of these field theories. Many new examples of AdS_3/CFT_2 in M-theory.
- The supergravity central charges match with the ones computed by anomalies and c-extremization for all twists with $(0,2)$ supersymmetry.
- Typically the SCFTs are "chiral", i.e. $c_r - c_\ell \neq 0$. Well-known for 2d CFTs coming from M5-branes. [Ganor], [Maldacena-Strominger-Witten], [Kraus-Larsen], ...

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There are three special twists for $\mathfrak{g}_i > 1$

- $z_1 = \pm 1$ and $z_2 = \mp 1$. The geometry in M theory is $CY_4 = T^*(\Sigma_{\mathfrak{g}_1}) \times T^*(\Sigma_{\mathfrak{g}_2})$. This is a $(2,2)$ twist with

$$c_r = c_\ell = (\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(4 d_G h_G + 3 r_G) \ .$$

This is an integer multiple of 3 for any simply laced G! Is this a σ-model on a CY target?

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- $z_1 = z_2 = \pm 1$. This twist preserves $(0,4)$ supersymmetry (different from the MSW CFT)

  $$c_r = 2(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(4 d_G h_G + 3 r_G) \ , \quad c_\ell = 4(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(2 d_G h_G + r_G) \ .$$

  Note that c_r is an integer multiple of 6 as appropriate for a $(0,4)$ theory.

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There are three special twists for $\mathfrak{g}_i > 1$

- $z_1 = \pm 1$ and $z_2 = \mp 1$. The geometry in M theory is $CY_4 = T^*(\Sigma_{\mathfrak{g}_1}) \times T^*(\Sigma_{\mathfrak{g}_2})$. This is a $(2,2)$ twist with

  $$c_r = c_\ell = (\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(4d_G h_G + 3r_G) \, .$$

  This is an integer multiple of 3 for any simply laced G! Is this a σ-model on a CY target?

- $z_1 = z_2 = \pm 1$. This twist preserves $(0,4)$ supersymmetry (different from the MSW CFT)

  $$c_r = 2(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(4d_G h_G + 3r_G) \, , \quad c_\ell = 4(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(2d_G h_G + r_G) \, .$$

  Note that c_r is an integer multiple of 6 as appropriate for a $(0,4)$ theory.

- $z_1 = z_2 = 0$. This is a special $(0,2)$ twist with $SU(2)_F \times U(1)_R$ global symmetry

  $$c_r = 3(\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(d_G h_G + r_G) \, , \quad c_\ell = (\mathfrak{g}_1 - 1)(\mathfrak{g}_2 - 1)(3d_G h_G + 2r_G) \, .$$

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Summary

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- Evidence for many new 2d $(0, 2)$ SCFTs, with explicit holographic duals, arising from wrapped D3-branes and M5-branes. Examples of the utility of c-extremization.
- Novel supersymmetric AdS_3 vacua of IIB and 11D supergravity.

Outlook

... still many things to understand

- Is there a 2d/2d correspondence à la AGT? What about 2d/4d correspondence? [Gadde-Gukov-Putrov]
- Understand punctures on the Riemann surface, i.e. flavor symmetries in the field theory. Is there a 2d analog of Gaiotto's T_N theories?
- Construct directly the two-dimensional $(0,2)$ SCFTs. Should be feasible for the ones coming from D3-branes. [Kapustin]
- Other applications of c-extremization (maybe in $(0,2)$ GLSMs)? Relation to c-theorem?
- Gravity dual of c-extremization? Seems to work in 3d gauged supergravity [Karndumri-Ó Colgáin]. Volume minimization for "generalized Sasaki-Einstein" manifolds? [Martelli-Sparks-Yau]

MERCI!