2d (0,2) Theories and F-theory

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(0,2) and String Theory

2d (0,2) theories define the heterotic worldsheet:

- UV $\xleftrightarrow[\text{IR-limit}]{}$ IR
  (0,2) GLSM — heterotic (0,2) SCFT
- In the geometric NLSM phase they thus define a target space geometry X_3 with vector bundle V_{het}:

$$\text{2d (0,2) theory} \xrightarrow[\text{string map}]{\text{heterotic}} \text{target space } (X_3, V_{\text{het}})$$

This talk introduces a reversed process:

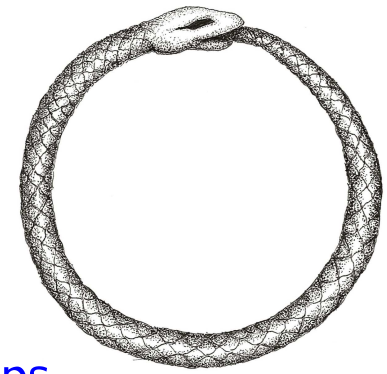
- We start with F-theory compactified on a torus-fibered Calabi-Yau 5-fold Y_5 with gauge flux G_4
- The effective 2d theory is a (0,2) theory.
- Thus here the geometry (Y_5, G_4) defines a 2d (0,2) theory:

$$\text{compactification space } (Y_5, G_4) \xrightarrow{\text{F-theory}} \text{2d (0,2) theory}$$

F-theory (0,2) engineering

Under suitable conditions the 2d (0,2) theory is a 2d (0,2) GLSM:

"Strings from strings"



- Is there a correspondence compactification geometry $\leftrightarrow$ target space?
- F-theory gives GLSM with general gauge groups - what is their target space interpretation? eg [Donagi,Sharpe'07],[Jockers,Kumar,Lapan,Morrison,Romo'12]

Does F-theory give a geometric interpretation of 2d (0,2) phenomena, e.g.

- non-abelian trialities? [Gadde,Gukov,Putrov'13]
- (0,2) mirror symmetry? e.g. [Melnikov,Plesser'10], . . . ; cf. talk by Anderson
- interpretation of elliptic genus as computed in [Gadde,Gukov] [Benini,Eager,Hori,Tachikawa]'13; [Israel,Sarkis'15] ?

The setup

F-theory on elliptic (torus) fibration Y_5 over B_4: effective theory in $\mathbb{R}^{1,1}$

$$\begin{array}{cccc} \pi: & \mathbb{E}_\tau & \rightarrow & Y_5 \\ & & & \downarrow \\ & & & B_4 \end{array}$$

Gauge degrees of freedom $\leftrightarrow$ 7-branes on Kähler 3-cycle M_G

Two different regimes:

1. Decoupling limit:
   - Vol$(B_4) \rightarrow \infty$, but Vol(M_G) finite
   - 2d $N = (0,2)$ gauge theory = (0,2) gauged linear sigma model (GLSM) with general gauge group from 7-branes
   - extra sector from D3-branes on complex curves
2. Coupling to 2d (0,2) supergravity:
   - Vol(B_4) finite
   - not yet investigated in full detail

The setup

This talk focuses on the gauge sector.

Two approaches of [Schäfer-Nameki,TW'16]

1. Twisted gauge theory on 7-branes 4d:[Beasley,Heckman,Vafa][Donagi,Wijnholt]'08
   - start from 8d SYM on 7-brane on $\mathbb{R}^{1,1} \times M_G$
   - perform partial topological twist to 2d $(0,2)$
   - matter at intersection of two 7-branes
     $\leftrightarrow$ coupling to 6d $N = (0,1)$ defect theory

2. Duality with M-theory on $\mathbb{R} \times Y_5$
   - $N = 2$ super-mechanics [Haupt,Lukas,Stelle'08]
   - explicit examples of fully consistent fibrations including fluxes:
     ✓ gauge anomalies
     ✓ Chern-Simons terms

The setup

Basic structure of 7-brane theory:

Singularities above codim	2d $N = (0, 2)$ Gauge Theory
1	Gauge algebra $\mathfrak{g}$
	Bulk matter+ interactions: E and J
2	Matter (chiral and Fermi) in $\mathbf{R} \oplus \bar{\mathbf{R}}$
	Bulk-surface matter couplings: E and J
3	Cubic matter couplings: E and J
4	Quartic matter couplings: E and J

First steps - many remaining directions to explore, in particular

- coupling to gravity
- interpretation of 2d scalar VEVs (Coleman-Mermin-Wagner theorem!)

Related work

- F-theory on 5-folds: [Apruzzi,Hassler,Heckman,Melnikov]: 1602.04221
- M-theory on general (non-fibered) 5-folds: [Haupt,Lukas,Stelle'08]
- Type II/heterotic on Calabi-Yau 4-folds:

  [Dasgupta,Mukhi'96] [Gukov,Vafa,Witten'99]

  [Gates,Gukov,Witten'00],[Haack,Louis,Marquart'00][Font,Lopez'04][Greiner,Grimm'15]
- 2d (0,2) theories from D1-branes at singularities in CY_4:

  [Mohri'97] [Garcia-Compean,Uranga'98] [Franco,Ghim,Lee,Seong,Yokoyama'15]

  [Franco,Lee,Seong'15/16]
- Major developments in understanding of F-theory of torus-fibered Calabi-Yau in past years
- A large literature on 2d (0,2) gauge theories

Plan

I) Introduction

II) Reminder: 2d (0,2) theories

III) Twisting 8d SYM

- Bulk theory
- Coupling to 6d defects (matter)

IV) CY 5-fold embedding

- couplings in codim 3 and 4
- D3-branes
- F/M duality, fluxes, tadpoles
- gauge anomalies
- Chern Simons-terms

V) Outlook

II) Reminder: 2d (0,2) theories

2d (0,2) SUSY

- 2 chiral supercharges with SUSY parameters ϵ_-, $\bar{\epsilon}_-$ [Witten'93]
- Superspace $(y^0, y^1; \theta^+, \bar{\theta}^+)$
- Matter content

Multiplet	Superfield	Content
gauge	V	$(v_0, v_1; \eta_-, \bar{\eta}_-; \mathfrak{D})$
chiral	Φ	$(\varphi; \chi_+; -)$
Fermi	P	$(-; \rho_-; E, G)$

- 2d gauge potential has no propagating degrees of freedom, but gauge symmetry acts as constraint
- Opposite chiralities: ρ_- versus χ_+
- Fermi auxiliary fields: G and $E = E(\Phi)$ holomorphic

2d (0,2) SUSY

Multiplet	Superfield	Content
gauge	V	$(v_0, v_1; \eta_-, \bar\eta_-; \mathfrak{D})$
chiral	Φ	$(\varphi; \chi_+; -)$
Fermi	P	$(-; \rho_-; E, G)$

2 types of holomorphic interactions [Witten'93]

- E-type interaction: $E_a(\Phi_i)$

$$L^F = -\frac{1}{2}\int d^2y d^2\theta\, P_a \bar{P}_a = -\int d^2y \left(\bar\rho_{-,a}\frac{\partial E_a}{\partial\varphi_i}\chi_{+,i} + \text{c.c.}\right) + \ldots$$

- Superpotential: $J^a(\Phi_i)$

$$L^J = -\frac{1}{\sqrt{2}}\int d^2y\, d\theta^+ \; P_a J^a(\Phi_i)|_{\bar\theta^+=0} - \text{c.c.} = -\int d^2y \left(G_a J^a + \rho_{-,a}\frac{\partial J^a}{\partial\varphi_i}\chi_{+,i} + \text{c.c.}\right)$$

subject to constraint $\mathrm{Tr} J^a(\Phi) E_a(\Phi) = 0$

2d (0,2) SUSY

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2 types of holomorphic interactions

- E-type interaction: $E_a(\Phi_i)$

$$L^F = -\frac{1}{2}\int d^2y d^2\theta \, P_a \bar{P}_a \supset -\int d^2y \left(\bar{\rho}_{-,a} \frac{\partial E_a}{\partial \varphi_i} \chi_{+,i} + c.c. \right) + \ldots$$

- Superpotential: $J^a(\Phi_i)$

$$L^J = -\frac{1}{\sqrt{2}}\int d^2y \, d\theta^+ \; P_a J^a(\Phi_i)|_{\bar{\theta}^+=0} - \text{c.c.} \supset -\int d^2y \left(G_a J^a + \rho_{-,a} \frac{\partial J^a}{\partial \varphi_i} \chi_{+,i} + \text{c.c.} \right)$$

Scalar potential $V = \frac{1}{2e^2}\mathfrak{D}^2 + \sum_a \left(|J^a|^2 + |E_a|^2 \right)$

III) Twisting 8d SYM

Twisting 8d SYM

- 8d SYM theory on a 7-brane with 16 supercharges $\mathbf{8^s}_{-1} + \mathbf{8^c}_1$
- Worldvolume: $\mathbb{R}^{1,1} \times M_G$ $\qquad$ M_G: Kähler 3-cycle
- Holonomy group of M_G: $SO(6)_L \supset U(3)_L = SU(3)_L \times U(1)_L$

$$\mathbf{8^c}_{+1} \longrightarrow [\mathbf{4} + \bar{\mathbf{4}}]_1 \longrightarrow [(\mathbf{3}_{-1} + \mathbf{1}_3) + (\bar{\mathbf{3}}_1 + \mathbf{1}_{-3})]_1$$

- Supersymmetry $\longleftrightarrow$ globally defined spinor (holonomy singlet)
- **Twisting** $=$
  redefine tangent bundle such that holonomy singlet exists [Vafa,Witten'94]

  Here:

$$U(1)_L \to U(1)_{\text{twist}} = \frac{1}{2}(U(1)_L + 3U(1)_R)$$

Twisting 8d SYM

- 8d SYM theory on a 7-brane with 16 supercharges $\mathbf{8^s}_{-1} + \mathbf{8^c}_{+1}$

$$SO(1,7)_L \times U(1)_R \quad \rightarrow \quad SU(3)_L \times SO(1,1)_L \times (U(1)_L \times U(1)_R)$$

$$\begin{array}{ccc} \mathbf{8^c}_{+1} & \rightarrow & \mathbf{1}_{1;3,1} \oplus \mathbf{1}_{-1;-3,1} \oplus \mathbf{3}_{1;-1,1} \oplus \overline{\mathbf{3}}_{-1;1,1} \\ \mathbf{8^s}_{-1} & \rightarrow & \mathbf{1}_{-1;3,-1} \oplus \mathbf{1}_{1;-3,-1} \oplus \mathbf{3}_{-1;-1,-1} \oplus \overline{\mathbf{3}}_{1;1,-1} \end{array}$$

- Twisting:

$$U(1)_L \rightarrow U(1)_{\rm twist} = \tfrac{1}{2}(U(1)_L + 3U(1)_R)$$

$$\bar{\epsilon}_- = \mathbf{1}_{-1;-3,1;0_{\rm twist}} \,, \qquad \epsilon_- = \mathbf{1}_{-1;3,-1;0_{\rm twist}} \Longrightarrow \text{2d (0,2)}$$

- Repeat decomposition for SYM matter: $\mathbf{8^v}_0$, $\mathbf{1}_{\pm 2}$, $\mathbf{8^c}_{+1}$, $\mathbf{8^s}_{-1}$

- Internal component w.r.t. $SU(3)_L \times U(1)_{\rm twist}$:

$$\mathbf{1}_0 \leftrightarrow \Omega^{0,0}(M_G) \quad \mathbf{3}_1 \leftrightarrow \Omega^{0,1}(M_G) \quad \overline{\mathbf{3}}_2 \leftrightarrow \Omega^{0,2}(M_G) \quad \mathbf{1}_3 \leftrightarrow \Omega^{0,3}(M_G)$$

Twisting 8d SYM

Massless modes counted by $H^{(0,p)}(M_G)$ plus conjugate [Schafer-Nameki,TW'16]

Cohomology	Bosons	Fermions	Multiplet
$H^{(0,0)}$	$v_\mu, \mu = 0, 1$	$\eta_-, \bar{\eta}_-$	Vector
$H^{(1,0)} \oplus H^{(0,1)}$	$\bar{a}, a$	$\bar{\psi}_+, \psi_+,$	Conjugate-chiral + Chiral (Wilson lines)
$H^{(2,0)} \oplus H^{(0,2)}$	$-$	$\rho_-, \bar{\rho}_-$	Fermi + Conjugate-Fermi
$H^{(3,0)} \oplus H^{(0,3)}$	$\varphi, \bar{\varphi}$	$\chi_+, \bar{\chi}_+$	Chiral + Conjugate-chiral (deformations of M_G)

Multiplet structure confirmed by SUSY variation

- in absence of gauge bundle all bulk matter in adjoint of G
- with gauge background

$$\mathbf{Adj}(G) \rightarrow \bigoplus_{\mathbf{R}} \mathbf{R}\,.$$

with matter in $\mathbf{R}$ counted by $H^{(0,p)}(M_G, L_{\mathbf{R}}) = H^p_{\bar{\partial}}(M_G, L_{\mathbf{R}})$

Twisting 8d SYM

- 10d/8d SUSY variation $\delta A_M = -i\bar{\epsilon}\Gamma_M \Psi, \qquad \delta\Psi = \frac{1}{2} F_{MN}\Gamma^{MN}\epsilon$
- Decomposition e.g. for fermions:

$$
\begin{aligned}
\delta\bar{\chi}_{+\bar{k}\bar{m}\bar{n}} &= -i\sqrt{2}\epsilon_- D_+ \bar{\varphi}_{\bar{k}\bar{m}\bar{n}} \\
\delta\psi_{+\bar{m}} &= i\sqrt{2}\bar{\epsilon}_- D_+ a_{\bar{m}} \\
\delta\eta_- &= \epsilon_-(F_{01} + i\mathfrak{D}) \qquad \mathfrak{D} = \frac{i}{2}\left(J\wedge J\wedge F_{M_G} + [\varphi, \overline{\varphi}]\right) \\
\delta\rho_{-mn} &= \epsilon_- \bar{F}_{mn} + \bar{\epsilon}_-(\partial_a^\dagger \varphi)_{mn}
\end{aligned}
$$

- BPS equations = Hitchin equations for Higgs bundle (A, φ) on M_G:

$F^{(2,0)} = F^{(0,2)} = 0$		superpotential J
$(\partial_a^\dagger \varphi)_{mn} = (\bar{\partial}_{\bar{a}}^\dagger \bar{\varphi})_{\bar{m}\bar{n}} = 0$	$\leftrightarrow$	E-auxiliary term E
$J\wedge J\wedge F + [\varphi, \overline{\varphi}] = 0$		D-term $\mathfrak{D}$

Twisting 8d SYM

Cubic 8d SYM interaction induces E and J-type cubic couplings

- Wilson line a – deformation mode φ:

  $$E^{(\rho_-^\alpha)} = -\mathbf{f}_{\alpha\mu\epsilon}\, \Phi^\mu A^\epsilon$$

  $$S = \mathbf{f}_{\alpha\mu\epsilon} \int d^2y\; \bar{\rho}_-^\alpha \left(\varphi^\mu\, \psi_+^\epsilon + \chi_+^\mu\, a^\epsilon \right) \qquad \mathbf{f}_{\alpha\mu\epsilon} = \int_{M_G} \hat{\bar{\rho}}_{\bar{k}\bar{m},\alpha} \wedge \left(\hat{\varphi}_{kmn,\mu} \wedge \hat{\psi}_{\bar{n},\epsilon} \right)$$

- Wilson line a – Wilson line a :

  $$J_{(\rho_-^\alpha)} = -\mathbf{g}_{\alpha\beta\gamma}\, A^\beta\, A^\gamma$$

  $$S = \mathbf{g}_{\alpha\beta\gamma} \int d^2y\; \rho_-^\alpha\, a^\beta\, \psi_+^\gamma \qquad \mathbf{g}_{\alpha\beta\gamma} = \int_{M_G} \tilde{\rho}_{kmn\bar{n},\alpha} \wedge \hat{a}_{\bar{k},\beta} \wedge \hat{\psi}_{\bar{m},\gamma}$$

- $\mathrm{Tr}\, E \cdot J = 0$ must be automatically satisfied due to SUSY

  $\leftrightarrow$ constraint on couplings

Coupling to 6d matter

- Two Kähler 3-cycles intersect over Kähler surface $S_{\mathbf{R}} = M_{G_1} \cap M_{G_2}$
- Extra massless matter in representations of $G_1 \times G_2$
- From perspective of M_{G_1}:
  $S_{\mathbf{R}}$ is a defect with trapped localised zero-modes
- In flat space: Defect theory is $N = (0,1)$ SYM on $\mathbb{R}^{1,5}$ with hypermultiplet in repr $\mathbf{R}$
- Decomposition of general Kähler surface:

$$\begin{array}{rcl} SO(1,5)_L \times SU(2)_R & \rightarrow & U(1)_R \times (SU(2) \times U(1)_L \times SO(1,1)_L) \\ \text{SUSY parameters}: (\bar{\mathbf{4}}, \mathbf{2}) & \rightarrow & (\mathbf{1}_{+1} \oplus \mathbf{1}_{-1}) \otimes (\mathbf{1}_{+1,-1} \oplus \mathbf{1}_{-1,-1} \oplus \mathbf{2}_{0,+1}) \end{array}$$

- Twist this theory in a manner compatible with bulk theory on M_G:

$$J_{\text{twist}} = J_{U(1)_L} - J_{U(1)_R}$$

$\bar{\epsilon}_- = \mathbf{1}_{+1,+1,-1}$, $\epsilon_- = \mathbf{1}_{-1,-1,-1} \Longrightarrow$ compatible with 2d (0,2)

Coupling to 6d matter

Repeat twisting for hypermultiplet: [Schafer-Nameki, TW'16]

$$SO(1,5)_L \times SU(2)_R \rightarrow SU(2)_L \times SO(1,1)_L \times U(1)_{\text{twist}}$$

$$\underbrace{(\mathbf{4}, 1_R)}_{\text{fermions}} \rightarrow \begin{cases} 1_{+1,-1} & \Omega^{0,0}(S_{\mathbf{R}}, \sqrt{K_S}) \\ \mathbf{2}_{-1,0} & \Omega^{0,1}(S_{\mathbf{R}}, \sqrt{K_S}) \\ 1_{+1,+1} & \Omega^{0,2}(S_{\mathbf{R}}, \sqrt{K_S}) \end{cases} \qquad \underbrace{(1, \mathbf{2}_R)}_{\text{scalars}} \rightarrow \begin{cases} 1_{+1,-1} & \Omega^{0,0}(S_{\mathbf{R}}, \sqrt{K_S}) \\ - & - \\ 1_{+1,+1} & \Omega^{0,2}(S_{\mathbf{R}}, \sqrt{K_S}) \end{cases}$$

- Zero-modes in presence of gauge bundle $L_{\mathbf{R}}$:

Cohomology	States	Multiplet
$H^0_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(T, \tau_+)^{\mathbf{R}}$	chiral
$H^1_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(-, \bar\mu_-)^{\mathbf{R}}$	conjugate Fermi
$H^2_{\bar\partial}(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}})$	$(\bar S, \bar\sigma_+)^{\mathbf{R}}$	conjugate chiral

- Kähler surface $S_{\mathbf{R}}$ not necessarily spin

  Flux quantization condition $\Longrightarrow$ $L_{\mathbf{R}} \otimes \sqrt{K_{S_{\mathbf{R}}}}$ is honest bundle on $S_{\mathbf{R}}$

Coupling to 6d matter

Holomorphic bulk couplings induce bulk-surface couplings

- View localised matter as trapped zero modes of bulk fields
- Structure of couplings:

$$\text{bulk}|_{S_{\mathbf{R}}} \times \text{surface} \times \text{surface}$$

$$\begin{aligned}
S_{\text{bulk+matter}} &= S^{(F)}_{\text{bulk+matter}} + S^{(J)}_{\text{bulk+matter}} \\
S^{(F)}_{\text{bulk+matter}} &= \mathbf{b}_{\alpha\beta\gamma} \int d^2y\, \overline{\rho}^{\alpha}_{-} \left(\tau^{\beta}_{+}\, S^{\gamma} + \sigma^{\gamma}_{+}\, T^{\beta} \right) + \text{c.c.} \\
&\quad + \mathbf{e}_{\delta\gamma\epsilon} \int d^2y\, \overline{\mu}^{\delta}_{-} \left(S^{\gamma}\, \psi^{\epsilon}_{+} + \sigma^{\gamma}_{+}\, a^{\epsilon} \right) + \text{c.c.} \\
S^{(J)}_{\text{bulk+matter}} &= \mathbf{c}_{\delta\beta\epsilon} \int d^2y\, \mu^{\delta}_{-} \left(T^{\beta}\, \psi^{\epsilon}_{+} + \tau^{\beta}_{+}\, a^{\epsilon} \right) + \text{c.c.}
\end{aligned}$$

- Corresponding modification of E-terms and J-superpotentials

  [Schafer-Nameki,TW'16]

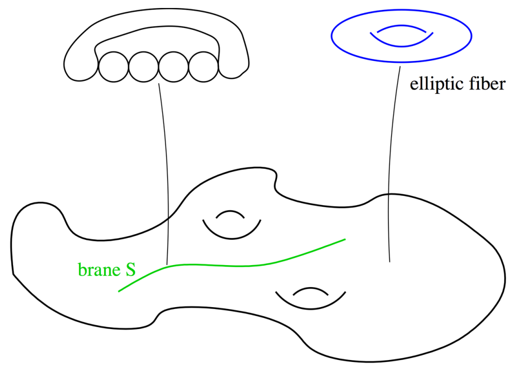
Summary so far

Twisting of 8d SYM coupled to 6d N=(0,1) defect:

Singularities above codim	2d $N = (0, 2)$ Gauge Theory
1	Gauge algebra $\mathfrak{g}$
	Bulk matter + interactions: E and J
2	Matter (chiral and Fermi) in $\mathbf{R} \oplus \bar{\mathbf{R}}$
	Bulk-surface matter couplings: E and J

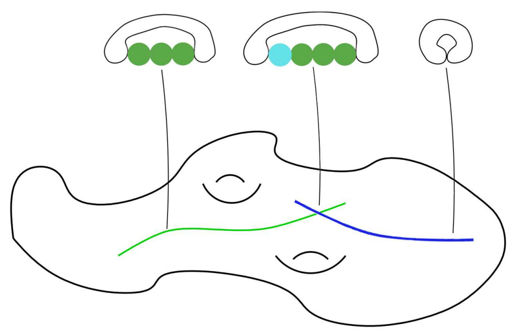
IV) CY 5-fold embedding

CY 5-fold embedding



7-brane on 3-cycle M_G — codimension one singularity, algebra $\mathfrak{g}$

$\mathbb{P}^1$s F_i above codim 1 loci M_G $\leftrightarrow$ Simple roots α_i of $\mathfrak{g}$



matter surface $S_{\mathbf{R}}$ — codimension two, fibre splitting

$\mathbb{P}^1$s above codim 2 loci $S_{\mathbf{R}}$ $\leftrightarrow$ Matter in Representation $\mathbf{R}$

$$F_i \rightarrow \underbrace{C_i^+}_{\text{weight of } \mathbf{R}} + \underbrace{C_{i+1}^-}_{\text{weight of } \bar{\mathbf{R}}}$$

CY 5-fold embedding

Further enhancements from fibre splittings in codimension three and four:

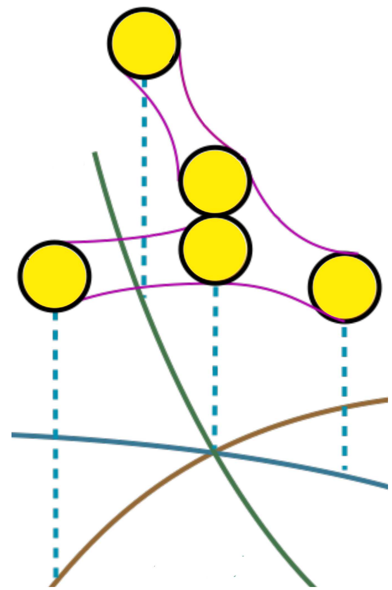
Codimension three curve

- $\Sigma_{\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3} = S_{\mathbf{R_1}} \cap S_{\mathbf{R_2}} \cap S_{\mathbf{R_3}}$
- fibre curve splitting:

$$C_{\lambda_{\mathbf{R}_1}} \to C_{\lambda_{\mathbf{R}_2}} + C_{\lambda_{\mathbf{R}_3}}$$

- gauge invariant contraction

$$\mathbf{R}_1 \oplus \mathbf{R}_2 \oplus \mathbf{R}_3 \;\to\; \mathbb{C}$$



cubic E and J-type couplings from wavefunction overlaps over $\Sigma_{\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3}$

[Schäfer-Nameki,TW'16] [Apruzzi,Hassler,Heckman,Melnikov'16]

$$J_{\left(\mu_-^{\mathbf{R}_1,\delta}\right)} = -\mathbf{h}_{\delta\epsilon\gamma}\left(\mathcal{Z}_2^{\mathbf{R}_2,\epsilon}\,\mathcal{Z}_3^{\mathbf{R}_3,\gamma}\right), \quad E^{\left(\mu_-^{\bar{\mathbf{R}}_2,\delta}\right)} = -\mathbf{d}_{\delta\epsilon\gamma}\left(\mathcal{Z}_1^{\mathbf{R}_1,\epsilon}\,\mathcal{Z}_3^{\mathbf{R}_3,\gamma}\right)$$

$\mathcal{Z} = \mathcal{S}$ or $\mathcal{T}$ - depending on which combination is gauge invariant

CY 5-fold embedding

Further enhancement points in base - codimension four

[Schäfer-Nameki,TW'16] [Apruzzi,Hassler,Heckman,Melnikov'16]

- At intersection of two coupling curves $\Sigma_{\mathbf{R}_i\mathbf{R}_j\mathbf{R}_j}$
- Structure of curve splitting in agreement with quartic couplings

$$\mathbf{R}_1 \oplus \mathbf{R}_2 \oplus \mathbf{R}_3 \ \oplus \mathbf{R}_4 \ \to \ \mathbb{C}$$

- In 2d scalar fields have mass dimension zero

  Couplings $(\text{fermion})^2 \times (\text{scalar})^2$ still 'super-renormalisable'
- Expect all combinations for J and E-type couplings in agreement with gauge invariance

CY 5-fold embedding

Example: $G = SU(5)$ in CY 5-fold

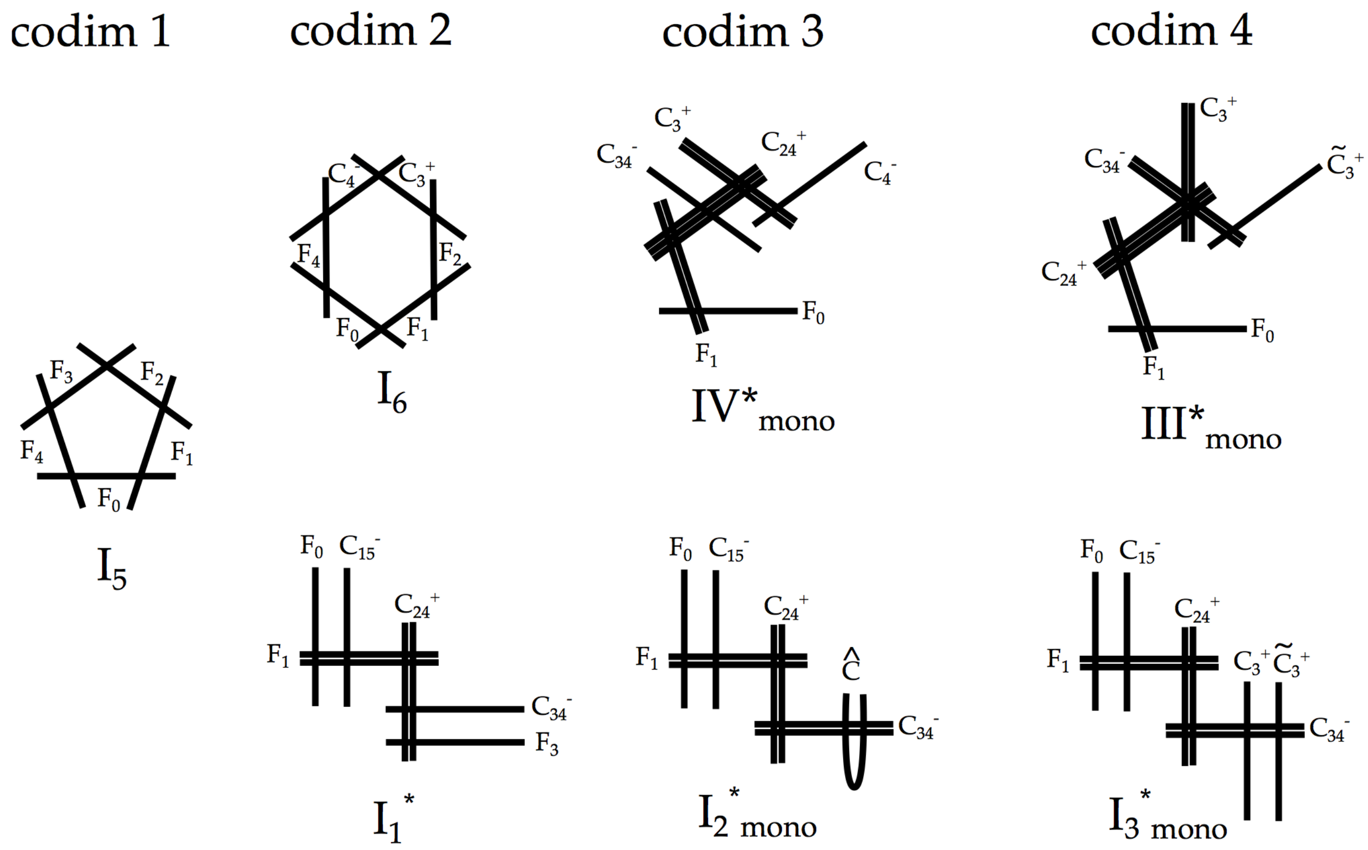
$$y^2 + b_1 xyz + b_3 w^2 yz^3 = x^3 + b_2 w x^2 z^2 + b_4 w^3 x z^4 + w^5 z^5 b_6 \,,$$

Discriminant at

$$\Delta = \quad w^5 \Big(b_1^4 \left(b_2 b_3^2 + b_1 b_3 b_4 + b_1^2 b_6 \right) + \mathcal{O}(w) \Big) \qquad w = 0 : \; SU(5) - \text{brane}$$

Codim 2 :	$SO(10)$:	$b_1 = 0$	$\mathbf{10}$ matter
	$SU(6)$:	$b_1^2 b_6 - b_1 b_3 b_4 + b_2 b_3^2 = 0$	$\mathbf{5}$ matter
Codim 3 :	$SO(12)$:	$b_1 = b_3 = 0$	$\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ coupling
	E_6 :	$b_1 = b_2 = 0$	$\mathbf{10}\,\mathbf{10}\,\mathbf{5}$ coupling
Codim 4 :	$SO(14)$:	$b_1 = b_3 = b_4^2 - 4 b_2 b_6 = 0$	$-$
	E_7 :	$b_1 = b_2 = b_3 = 0$	$(\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} + \mathbf{10}\,\mathbf{5}\,\mathbf{5}\,\mathbf{5})$ couplings

CY 5-fold embedding



Many more examples in [Schäfer-Nameki,TW'16]

- including with exceptional gauge algebra

D3-brane sector

D3-branes wrap curves C^B on base B_4

- Dictated by tadpole cancellation condition (see later)
- gauge coupling $\frac{1}{g^2_{\rm D3}} = e^{-\phi}\,{\rm Vol}(C^B_{\rm M2})\,\ell_s^2$

So far, only perturbative treatment:

- Intersection $C^B \cap M_G =$ points
- D3-7 strings 8 DN boundary conditions $\Rightarrow a_{NS} = -\frac{1}{2} + \frac{8}{8} = \frac{1}{2}$
- Only 1 fermionic degree of freedom

  $\Rightarrow$ Fermi multiplet ν_- in fundamental of 7-brane group G
- Number of Fermi multiplets ν_-: [Schäfer-Nameki, TW'16]

$$\frac{1}{{\rm ord}(g)} \times \int_{B_4} [M_G] \wedge [C^B_{\rm M2}] \qquad g: \text{extra monodromy group}$$

Relevance: D3-7 states contribute to gauge anomalies!

Global embedding

Tadpoles and fluxes incorporated via duality with M-theory:

$$\begin{array}{ccc} \text{M-theory on } Y_5 & \xrightarrow{\text{Vol}(\mathbb{E}_\tau)\to 0} & \text{F-theory on } Y_5 \\ \downarrow & & \downarrow \\ \text{1d Super-Mechanics} & \xrightarrow{R_A \sim \frac{1}{R_B} \to 0} & \text{2d } (0,2) \text{ Theory} \end{array}$$

Gauge background $\leftrightarrow$ 4-form flux G_4 subject to

- transversality condition [Dasgupta,Rajesh,Sethi'99]
  $\int_{Y_5} G_4 \wedge S_0 \wedge \omega_4 = 0 \qquad \int_{Y_5} G_4 \wedge \omega_6 = 0 \qquad \forall\, \omega_4 \in H^4(B_4), \quad \omega_6 \in H^6(B_4)$
- quantisation condition [Witten'96]
  $G_4 + \frac{1}{2} c_2(Y_5) \in H^4(Y_5, \mathbb{Z})$
- F-term condition $G_4 \in H^{2,2}(Y_5)$
- D-term condition* $\int_{Y_5} G_4 \wedge J \wedge J \wedge J = 0$

Global embedding

D3-brane $\leftrightarrow$ M2-brane along $\mathbb{R} \times C^B_{\rm M2}$

$$S_{\rm M} = -2\pi \int_{\mathbb{R}\times Y_5} \frac{1}{2} G_4 \wedge *G_4 + C_3 \wedge \left(\delta([C_{\rm M2}]) - \frac{1}{24} c_4(Y_5) + \frac{1}{6} G_4 \wedge G_4 \right)$$

M2-tadpole: [Haupt,Lukas,Stelle'08]

$$\delta([C_{\rm M2}]) = \tfrac{1}{24} c_4(Y_5) - \tfrac{1}{2} G_4 \wedge G_4 \in H^8(Y_5)$$

Base components $\leftrightarrow$ spacetime-filling D3-brane tadpole in F-theory:

$$[C^B_{\rm M2}] = \frac{1}{24}[c_4(Y_5)]_B - \frac{1}{2}[G_4 \wedge G_4]_B \underbrace{\overset{!}{\geq}}_{\text{SUSY!}} 0$$

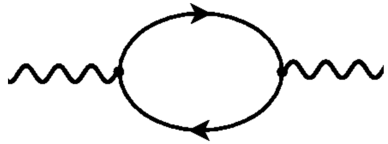
Number of 3-7-brane Fermi multiplets:

$$\# = \frac{1}{\mathrm{ord}(g)} M_G \cdot_{B_4} [C^B_{\rm M2}] = \frac{1}{\mathrm{ord}(g)} M_G \cdot_{B_4} \left(\frac{1}{24}[c_4(Y_5)]_B - \frac{1}{2}[G_4 \wedge G_4]_B \right)$$

Gauge anomalies

2d anomalies are quadratic:

$$\partial_\mu J^{\mu a} = \frac{1}{8\pi}\,\mathrm{Tr}(\gamma^3 T^a_{\mathbf{R}} T^b_{\mathbf{R}})\, F^b_{\mu\nu}\epsilon^{\mu\nu}$$



Anomaly coefficient

$$\mathcal{A}(\mathbf{R},P) = P\,C(\mathbf{R}), \quad \mathrm{tr}\, T^a_{\mathbf{R}} T^b_{\mathbf{R}} = C(\mathbf{R})\,\delta^{ab}, \qquad P = \begin{cases} +1 & \text{chiral mult.} \\ -1 & \text{vector \& Fermi} \end{cases}$$

$$\mathcal{A}_{\text{total}} = \mathcal{A}_{\text{bulk}} + \mathcal{A}_{\text{surface}} + \mathcal{A}_{3-7}$$

- Total anomaly for non-abelian gauge group must vanish
- Guaranteed by anomaly inflow argument via D3-tadpole cancellation condition
- Has been checked explicitly in many examples with and without gauge flux [Schäfer-Nameki,TW'16]

Gauge anomalies

$$\mathcal{A}_{\text{non-ab.}} = \mathcal{A}_{\text{bulk}} + \mathcal{A}_{\text{surface}} + \mathcal{A}_{3-7} = 0$$

1) $\mathcal{A}_{\text{bulk}}$

$$\begin{aligned} \mathcal{A}_{\text{bulk}} &= \sum_{\mathbf{R}} C(\mathbf{R}) \sum_{i=0}^{3} (-1)^{i+1} h^i(M_G, L_{\mathbf{R}}) \\ &= -\sum_{\mathbf{R}} C(\mathbf{R})\, \chi(M_G, \mathbf{R}) \end{aligned}$$

Cohomology	Fermions
$H^0(M_G, L_{\mathbf{R}})$	$\bar{\eta}_-$
$H^1(M_G, L_{\mathbf{R}})$	ψ_+
$H^2(M_G, L_{\mathbf{R}})$	$\bar{\rho}_-$
$H^3(M_G, L_{\mathbf{R}})$	$\bar{\chi}_+$

2) $\mathcal{A}_{\text{S}}$

$$\begin{aligned} \mathcal{A}_{\text{S}} &= \sum_{\mathbf{R}} C(\mathbf{R}) \sum_{i=0}^{2} (-1)^{i} h^i(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_S}) \\ &= \sum_{\mathbf{R}} C(\mathbf{R})\, \chi(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_S}) \end{aligned}$$

Cohomology	Fermions
$H^0(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_S})$	τ_+
$H^1(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_S})$	$\bar{\mu}_-$
$H^2(S_{\mathbf{R}}, L_{\mathbf{R}} \otimes \sqrt{K_S})$	$\bar{\sigma}_+$

3) $$\mathcal{A}_{3-7} = -\frac{1}{\text{ord}(g)} M_G \cdot \left(\frac{1}{24}[c_4(Y_5)]_B - \frac{1}{2}[G_4 \wedge G_4]_B \right)$$

1d Chern-Simons couplings

$$S_{\mathrm{M}} = -2\pi \int_{\mathbb{R}\times Y_5} \frac{1}{2} G_4 \wedge * G_4 + C_3 \wedge \left(\delta([C_{\mathrm{M2}}]) - \frac{1}{24} c_4(Y_5) + \frac{1}{6} G_4 \wedge G_4 \right)$$

- Decompose $C_3 = \sum A^\alpha \wedge \omega_\alpha$, $\qquad \alpha = 1, \ldots, h^{1,1}(Y_5)$
- 1d Chern-Simons couplings:

$$S_{\mathrm{top}} = 2\pi \sum_\alpha \int_{\mathbb{R}} A_\alpha \wedge (k^\alpha_{\mathrm{M2}} + k^\alpha_{\mathrm{curv}})$$

$$k^\alpha_{\mathrm{M2}} = -\int_{Y_5} \omega_\alpha \wedge \delta([C_{\mathrm{M2}}]) \qquad k^\alpha_{\mathrm{curv}} = \int_{Y_5} \omega_\alpha \wedge \left(\frac{1}{24} [c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right)$$

- base classes k^a, $a = 1, \ldots h^{1,1}(B_4) \leftrightarrow$ F-theory D3-tadpole
- fibral classes k^i_{curv}, $i = 1, \ldots \mathrm{rk}(\mathfrak{g})$: $\omega_i \leftrightarrow$ resolution divisors D_i

$$\text{1d from M-theory}: \ \mathbf{k^i_{\mathrm{curv}}} \qquad \longleftrightarrow \qquad \text{2d from F-theory on } \mathbf{S^1}: \ \mathbf{k^i_{1-\mathrm{loop}}}$$

1d Chern-Simons couplings

$$S_{\rm M} = -2\pi \int_{\mathbb{R}\times Y_5} \frac{1}{2} G_4 \wedge * G_4 + C_3 \wedge \left(\delta([C_{\rm M2}]) - \frac{1}{24} c_4(Y_5) + \frac{1}{6} G_4 \wedge G_4 \right)$$

- 1d Chern-Simons couplings:

$$S_{\rm top} = 2\pi \sum_\alpha \int_{\mathbb{R}} A_\alpha \wedge (k^\alpha_{\rm M2} + k^\alpha_{\rm curv}) \qquad C_3 = \sum A^\alpha \wedge \omega_\alpha$$

$$k^\alpha_{\rm M2} = -\int_{Y_5} \omega_\alpha \wedge \delta([C_{\rm M2}]) \qquad k^\alpha_{\rm curv} = \int_{Y_5} \omega_\alpha \wedge \left(\frac{1}{24}[c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right)$$

- fibral classes $k^i_{\rm curv}$, $i = 1, \dots {\rm rk}(\mathfrak{g})$: $\omega_i \leftrightarrow$ resolution divisors D_i

1d from M-theory : $\mathbf{k^i_{\rm curv}}$ $\longleftrightarrow$ 2d from F-theory on $\mathbf{S^1}$: $\mathbf{k^i_{1-{\rm loop}}}$

5d/6d: [Witten'96] [Intriligator,Morrison,Seiberg'97] [Bonetti,Grimm,(Hohenegger)'11(13)] [Grimm,Kapfer,Keitel'13]

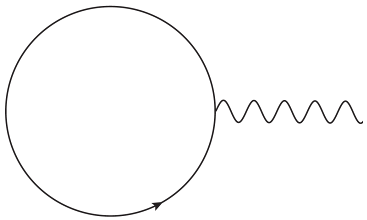
3d/4d: [Aharony et al.'97] [Grimm,Hayashi'11] [Cvetič,Grimm,Klevers'12]

1d Chern-Simons couplings

- CY_5 resolution = M-theory on Coulomb branch
- F-theory on S^1: KK zero mode + KK-tower
  - $m_0(\lambda_a^{\mathbf{R}}) = \sum_{j=1}^{\text{rk}(\mathfrak{g})} q_{aj} \langle \xi_j \rangle$ $\qquad q_{ai} = \lambda_{ai}^{\mathbf{R}} = (\pm_a)\ D_i \cdot_{Y_5} C^{\pm_a}_{\lambda_a^{\mathbf{R}}}$
  - $m_n(\lambda_a^{\mathbf{R}}) = m_0(\lambda_a^{\mathbf{R}}) + n \int_{\mathfrak{F}} J$
- Contribution of particle to CS-term

  1-loop exact [Witten'96]

  $\delta k^i_{1-\text{loop}} = -\frac{1}{2} P \sum_{a=1}^{\dim(\mathbf{R})} q_{ai}\, \text{sign}(m(\lambda_a^{\mathbf{R}}))$



- For elliptic fibration:

  KK non-zero-modes cancel and $\text{sign}\left(m_0(\lambda_a^{\mathbf{R}})\right) = \pm_a$

$$\underbrace{D_i \cdot_{Y_5} \left(\frac{1}{24}[c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4\right)}_{k^i_{\text{curv}}} = \underbrace{-\frac{1}{2} \sum_{\mathbf{R}} \left(n^+_{\mathbf{R}} - n^-_{\mathbf{R}}\right) \left(\sum_{a=1}^{\dim(\mathbf{R})} D_i \cdot_{Y_5} C^{\pm_a}_{\lambda_a^{\mathbf{R}}}\right)}_{k^i_{1-\text{loop}}}$$

1d Chern-Simons couplings

[Schäfer-Nameki,TW'16]

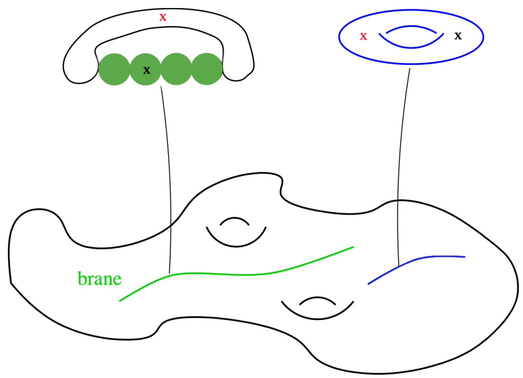
$$\underbrace{D_i \cdot_{Y_5} \left(\frac{1}{24}[c_4(Y_5)] - \frac{1}{2} G_4 \wedge G_4 \right)}_{k^i_{\text{curv}}} = \underbrace{-\frac{1}{2} \sum_{\mathbf{R}} \left(n^+_{\mathbf{R}} - n^-_{\mathbf{R}} \right) \left(\sum_{a=1}^{\dim(\mathbf{R})} D_i \cdot_{Y_5} C^{\pm_a}_{\lambda^{\mathbf{R}}_a} \right)}_{k^i_{1-\text{loop}}}$$

$$n^+_{\mathbf{R}} - n^-_{\mathbf{R}} = \begin{cases} -\chi(M_G, \mathbf{R}) & \text{bulk} \quad \text{matter} \\ +\chi(S_{\mathbf{R}}, \mathbf{R}) & \text{surface} \quad \text{matter} \end{cases}$$

Significance:

- for smooth matter loci $-\chi(M_G, \mathbf{R})$ and $\chi(S_{\mathbf{R}}, \mathbf{R})$ computable via Riemann-Roch
  $\rightarrow$ explicit consistency check of entire framework and elliptic fibrations
- for singular matter loci: normalization required for $\chi(S_{\mathbf{R}}, \mathbf{R})$
  CS terms give alternative to derive the chiral index

Inclusion of U(1)s



Extra rational sections σ_m of Y_5 [Morrison,Vafa'96]

- Shioda-map

  $S_m = \sigma_m - \sigma_0 - D_B - \sum_i n_i D_i$

- $C_3 = A_m \wedge S_m + \ldots$ A_m: $U(1)_m$ gauge field

New features:

- Purely local approach via Higgs bundles harder to argue due to matter surfaces away from non-abelian gauge branes

  Decoupling of gravity still possible

- Rich set of Green-Schwarz terms to cancel $U(1)$ gauge anomalies:

  [Mohri'96][Compean,Uranga'97] [Adams'09]; [Quigley,Sethi] [Blasczyk,Nibbelink,Ruehle]'11

  massive versus anomalous U(1)s

- FI-term: $r_m = \int_{Y_5} G_4 \wedge S_m \wedge J_B \wedge J_B$

V) Outlook

Outlook

So far have studied 2d $N = (0,2)$ gauge theory via

- topological twist
- F/M-theory duality

Codim	2d $N = (0,2)$ Gauge Theory
1	Gauge algebra $\mathfrak{g}$
2	Matter in $\mathbf{R} \oplus \bar{\mathbf{R}}$
	Bulk-surface couplings.: E and J
3	Matter couplings: E and J
4	Matter couplings: E and J

Many directions to explore, e.g.

- RG-flow to (0,2) SCFTs in the infra-red
  Geometrisation of RG flow by shrinking brane cycle volumes?
- Better non-perturbative understanding of D3-brane sector
- Full inclusion of 2d (0,2) SUGRA sector, grav. anomalies
- Worldsheet interpretation of 2d theory?
  non-abelian gauge theories — GLSM phases?

Outlook

Example: $(0,2)$ Quintic GLSM gauge group $U(1)$ gravity decoupled

$$E_i = \Phi_i\,\Sigma \qquad E_0 = \Phi_0\,\Sigma$$

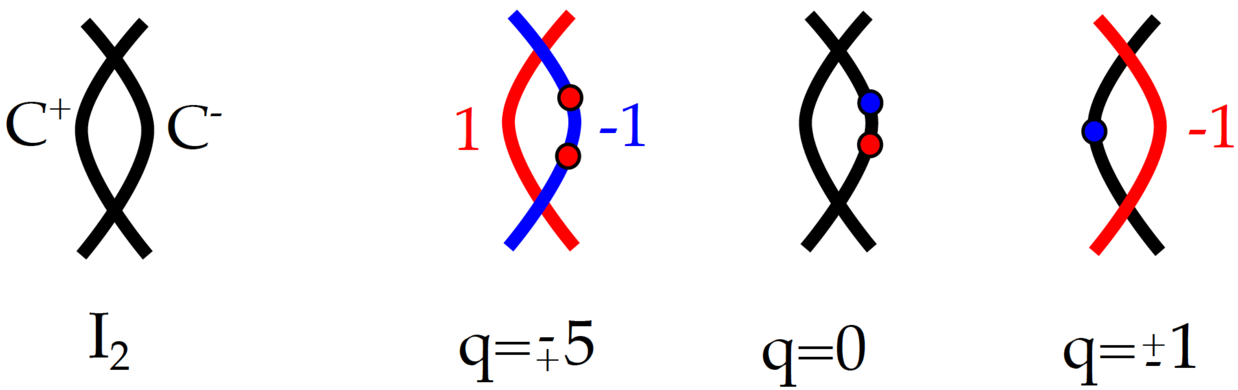
$$J^i = \Phi_0\,\mathcal{J}^i(\Phi_j) \qquad J^0 = \mathcal{P}(\Phi_j)\,,$$

$$V = V_F + V_D$$

$$V_D = \frac{e^2}{2}\left(\sum_i |\varphi_i|^2 - 5|\varphi_0|^2 - r\right)^2$$

Field	Type	$U(1)$ Charge
$\Phi_i,\ i = 1,\cdots,5$	Chiral	$+1$
$P_i,\ i = 1,\cdots,5$	Fermi	$+1$
Φ_0	Chiral	-5
P_0	Fermi	-5
Σ	Chiral	0

Required:



- fibre structure
- Flux background inducing correct multiplicities

Outlook

Interpreting GLSM as UV theory for $(0,2)$ NLSM suggests correspondence

$$(X_{\text{het}}, V_{\text{het}}) \quad \xleftrightarrow{\text{F-theory}} \quad (Y_5, G_4)$$

Caveat: In presence of gravity must integrate over all non-fixed scalar VEVs in 2d due to Coleman-Mermin-Wagner theorem

[Vafa, p.c.] [Apruzzi,Hassler,Heckman,Melnikov'16]

Solution: Take suitable decoupling limit with an effectively constant FI-parameter

Different values of FI terms correspond to different 2d regimes:

[Schäfer-Nameki,TW'16]

$$\begin{array}{ccccc} \text{NLSM} - \text{phase } r \gg 0 & & \text{GLSM} & & \text{LG} - \text{phase } r \ll 0 \\ G = \emptyset & \xleftarrow[\text{data}]{\text{gluing}} & G = U(1) & \xrightarrow[\text{data}]{\text{gluing}} & G = \mathbb{Z}_5 \\ (\tilde{A}, \tilde{\Phi}) & & (A, \Phi) & & (\hat{A}, \hat{\Phi}) \end{array}$$