(2,2) Abelian GLSMs, String Vacua, and Mirror Symmetry

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Outline

Mirror Symmetry is Deep Geometry?

- MS is a property of (2,2) SCFTs – but it is trivial in SCFT!
- Mirror Symmetry is a property of Calabi–Yau spaces. Some CYs are mirror pairs Which??
- Combinatorial duality for CICY in Gorenstein toric varieties
- In string theory, follow mirror pairs through extremal transitions: (almost) all CY spaces should have a mirror
- Mirror may not be geometrical?

Mirror Symmetry is Abelian Duality?

- Toric models have a UV free description in terms of Abelian GLSMs
- Natural map between toric parameter spaces
- Mirror symmetry is a T-duality of Abelian GLSM
- Abelian GLSMs are mirror pairs Which??
- Some Abelian GLSMs correspond to CY spaces Which??
- Geometry is a side effect in special examples?

Plan

- Describe the combinatorial data determining a family of GLSMs
- Find conditions under which the IR limit determines a family of $\mathcal{N} = (2,2)$ SCFTs with integral $U(1)$ charge
- Look for conditions under which the family is generically nonsingular and check for MS
- Reflexivity, which guarantees a geometric interpretation, is a sufficient condition Batyrev, Borisov; Batyrev, Nill
- Extremal transitions relate reflexive to non-reflexive models
- Formulate a weaker condition which we conjecture is sufficient, but not necessary
- Discuss some interesting examples

A Family of GLSMs

We define a family of $\mathcal{N} = (2,2)$ Abelian GLSMs by:

- A collection of n chiral superfields Φ_α
- A gauge group G acting diagonally and effectively via $Q : G \to \mathrm{U}(1)^n$ inducing $\mathfrak{q} : \mathfrak{g} \to \mathbb{R}^n$. For $G_{\mathbb{C}} = (\mathbb{C}^*)^r \times \Gamma$ we have r vector superfields $V \in \mathfrak{g}$ with invariant field strength $\Sigma = \frac{1}{\sqrt{2}}\overline{D}_+ D_- V$
- A family of gauge-invariant polynomial superpotentials $W(\Phi)$ determined by a collection of m invariant monomials $M_\beta = \prod_\alpha \Phi_\alpha^{p_{\beta\alpha}}$

The Lagrangian is

$$
\begin{aligned}
\mathcal{L} \;=\; & \int d^4\theta \left(\sum_{\alpha} e^{2\mathfrak{q}_\alpha(V)} |\Phi_\alpha|^2 - \frac{1}{4e^2} \|\Sigma\|^2 \right) \\
& + \left(\int d\theta^+ d\theta^- W(\Phi) + \text{ c.c. } \right) \\
& + \left(\frac{i}{\sqrt{2}} \int d\theta^+ d\bar{\theta}^- \langle \tau, \Sigma \rangle + \text{ c.c. } \right),
\end{aligned}
\tag{1}
$$

Parameters are

- Coefficients b_β in W (not renormalized)
- $\tau = \frac{\theta}{2\pi} + i\rho$ or $q = e^{2\pi i \tau}$ (renormalized at one-loop)

The Pointset $\mathscr{A}$

- Diagonal action gives a map $q : G \to (\mathbb{C}^*)^n$, taking $\mathrm{Hom}(-, \mathbb{C}^*)$
- Get a presentation of $\widehat{G} = \mathrm{Hom}(G_{\mathbb{C}}, \mathbb{C}^*) \sim \mathbb{Z}^r \oplus \Gamma$
-
$$0 \longrightarrow M \xrightarrow{A^t} \mathbb{Z}^n \xrightarrow{Q} \widehat{G} \longrightarrow 0, \tag{2}$$
  $M \sim \mathbb{Z}^d$, $A \in \mathrm{Mat}_{d\times n}(\mathbb{Z})$, $d = n - r$, and (free part of) Q is the charge matrix
- Columns of A determine a collection $\mathscr{A}$ of n points α in $N = M^\vee$
- With $T_N = N \otimes_{\mathbb{Z}} \mathbb{C}^*$ recover familiar
$$1 \longrightarrow G \xrightarrow{Q^t} (\mathbb{C}^*)^{\mathscr{A}} \xrightarrow{A} T_N \longrightarrow 1. \tag{3}$$

The Pointset $\mathscr{B}$

- The map $P : \mathbb{Z}^m \to \mathbb{Z}^n$ factors through the kernel of Q defining

$$B : \mathbb{Z}^m \to M, \tag{4}$$

with $P = A^t B$

- Columns of B determine a collection $\mathscr{B}$ of m points β in M with

$$W = \sum_{\beta \in \mathscr{B}} b_\beta \prod_\alpha \Phi_\alpha^{\langle \beta, \alpha \rangle} \tag{5}$$

- This is polynomial iff

$$\mathscr{B} \subset \mathrm{Cone}(\mathrm{Conv}\,\mathscr{A})^\vee. \tag{6}$$

The R-symmetry

- In general, model becomes strongly coupled at low energy and IR dynamics is trivial
- In $\mathcal{N} = (2,2)$ models, a nonanomalous chiral $\mathrm{U}(1)_R$ symmetry implies nontrivial IR dynamics
- Our superpotential admits an R symmetry under which gauge invariant chiral operators have integral charge iff

$$\exists \nu \in N : \langle \beta, \nu \rangle = 1, \quad \forall \beta \in \mathscr{B}. \tag{7}$$

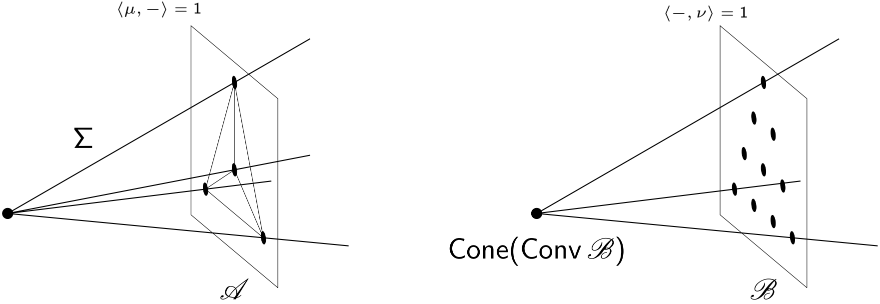
$\mathscr{B}$ lies in a primitive hyperplane

- The symmetry will be non-anomalous, and twisted sectors under Γ will have integral charge, iff

$$\exists \mu \in M : \langle \mu, \alpha \rangle = 1, \quad \forall \alpha \in \mathscr{A} \tag{8}$$

$\mathscr{A}$ lies in a primitive hyperplane

GLSM Data



$\langle \mu, - \rangle = 1$
Σ
$\mathscr{A}$
$\langle -, \nu \rangle = 1$
Cone(Conv $\mathscr{B}$)
$\mathscr{B}$

Mirror Symmetry

- If $\langle \mu, \mathscr{A} \rangle = \langle \mathscr{B}, \nu \rangle = 1$ IR dynamics governed by $\mathcal{N} = (2,2)$ SCFT with $c = 3(d - 2s)$ where $d = n - r$; $s = \langle \mu, \nu \rangle$.
- Exchanging $\mathscr{A}$ with $\mathscr{B}$ yields a mirror model. Note our conditions respect the symmetry – with the exception of

$$\mathscr{B} \subset \mathrm{Cone}(\mathrm{Conv}\, \mathscr{A})^\vee \tag{9}$$

- The pointsets are reflexive if

$$\mathrm{Cone}(\mathrm{Conv}\, \mathscr{A})^\vee = \mathrm{Cone}(\mathrm{Conv}\, \mathscr{B}). \tag{10}$$

Reflexive models come in mirror pairs

Parameter Space

- Varying our parameters is a deformation in well-known conformal manifold
- Coefficients $b \in \mathbb{C}^m$ parameterize chiral deformations redundantly. Field redefinitions $b_\beta \mapsto \prod_\alpha \lambda_\alpha^{p_{\beta\alpha}}$ produce a $(\mathbb{C}^*)^d$ identification
- Space of inequivalent models compactifies to toric variety $\mathscr{M}_B$ associated to secondary fan of $\mathscr{B}$.
- $\tau \in \mathfrak{g}^*$ are local coordinates on toric variety $\mathscr{M}_A$ associated to secondary fan of $\mathscr{A}$, with homogeneous coordinates a_α and $q_a = \prod_\alpha a_\alpha^{Q_a^\alpha}$ invariant under $a_\alpha \mapsto \prod_\beta \chi_\beta^{p_{\beta\alpha}}$
- Natural mirror map of parameter spaces exchanges a_α and b_β.

We study the parameter space $\mathscr{M}_A \times \mathscr{M}_B$. This is not the conformal manifold, because

- Our choice for $\mathscr{B}$ might omit some invariant monomials
- In general there are deformations of the SCFT that are not manifest in the GLSM (non-polynomial deformations)
- In general this is a redundant parameterization, there can be additional continuous identifications from nonlinear field redefinitions
- There are points in $\mathscr{M}_B$ that do not correspond to a SCFT but to singular limits.
- Same applies to $\mathscr{M}_A$.

A good family is one where the generic model is a SCFT. This is the last condition to be imposed

Classical Vacua

Classical SUSY vacua are zeroes of scalar potential

$$\begin{aligned} U(x,\sigma) &= |D|^2 + \sum_\alpha |F_\alpha|^2 + \sigma^\dagger M(\phi)\sigma \\ D &= \sum_\alpha Q_\alpha |\phi_\alpha|^2 - \rho \in \mathfrak{g}_{\mathbb{R}}^* \\ F_\alpha &= \frac{\partial W}{\partial \phi_\alpha} \\ M &= \sum_\alpha {}^t Q_\alpha Q_\alpha |\phi_\alpha|^2 \in \mathfrak{g}_{\mathbb{R}}^{\otimes 2} \end{aligned} \tag{11}$$

Solutions related by the action of G via Q are identified.

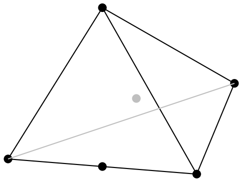
Classical Vacua

- Values of ρ for which M has a kernel in the space of solutions to (11) lie in cones and form the faces of a fan in $\mathfrak{g}_{\mathbb{R}}^* = \mathbb{R}^r$, the secondary fan of $\mathscr{A}$
- Large cones in this fan are associated to a choice of triangulation of $\mathscr{A}$ which, in turn, determines a fan Σ
- Σ gives an irrelevant ideal B in $S = \mathbb{C}[\phi]$ for ρ in the associated large cone and we get the toric variety
- $$Z_\Sigma = \frac{\operatorname{Spec} S - V(B)}{G}. \qquad (12)$$

  is non-compact and $K_Z = 0$. Need not be Gorenstein
- Space of vacua is then $\operatorname{Crit}(W) \subset Z_\Sigma$
- In general there are massless chiral fields interacting via a superpotential - bad hybrid
- The classical description is valid for ρ deep in the interior of a large cone (phase limit)

A Familiar Example - $\mathbb{P}^4_{1,1,2,2,2}[8]$

$\mathscr{A}$ has 7 points, $d = 5$. Five points are vertices of a simplex, one point is in the interior of this, and one along an edge.



There are four phases (triangulations). Using all 7 points, Z_Σ is a line bundle over resolved $\mathbb{P}^4_{1,1,2,2,2}$

$\mathscr{B}$ has 105 points (degree-8 monomials) lying in a simplex

In mirror model $\mathscr{B}$ has 7 points, superpotential has 7 terms in 105 variables. In LG phase, in which we use only vertices of $\mathscr{A}$

$$W = b_1 x_1^4 + b_2 x_2^4 + b_3 x_3^4 + b_4 x_4^8 + b_5 x_5^8 + b_6 x_4^4 x_5^4 + b_7 x_1 x_2 x_3 x_4 x_5. \tag{13}$$

Singularities

- If Crit W has a noncompact component extending to infinity, semiclassical considerations are valid for these vacua and show a continuum of states extending to zero energy, leading to singular low-energy behavior
- Values of ρ in faces of the secondary fan lead to solutions for ϕ for which $M(\phi)$ has a kernel. A continuous subgroup of G is unbroken and associated σ is free, introducing a noncompact component to space of vacua extending to infinity and leading to singular IR dynamics
- There are quantum corrections to this but large-field behavior is very well controlled
- Topological models, where semiclassical limit is exact, show there are no additional singularities, so
- Model is nonsingular if space of (semi-) classical vacua is compact.

Geometry?

- In some phase, Crit W may be a CY variety and SCFT is low-energy limit of NLSM on this for some parameters.
- When does a family include a CY phase?
- In the reflexive case, if $\mathscr{A}$ contains an $s-1$-simplex such that each facet of Conv $\mathscr{A}$ contains $s-1$ of its vertices – a special simplex) – and the same for $\mathscr{B}$, then there is a phase in which Z_Σ is a Gorenstein CY and Crit W is a complete intersection of s hypersurfaces and CY. Exchanging $\mathscr{A}$ and $\mathscr{B}$ get a mirror pair of these Batyrev Borisov; Batyrev, Nill
- There are models for which $\mathscr{A}$ admits a special simplex but $\mathscr{B}$ does not; in this case we have only one geometrical interpretation (Z orbifold; Aspinwall, Greene)
- Can force a geometry by adding $\nu \in N$ to $\mathscr{A}$. For $s > 1$ this leads to a non-CY space

Generic Orbits

- Assume q_a sufficiently generic that all σ massive, then space of vacua given by expectation values of ϕ lying in $X_\Sigma \subset Z_\sigma$
- In fact, singular values of b independent of q so need not choose a phase
- R-symmetry means $\lambda^\nu : \mathbb{C}^* \to T_N$ given by $\nu \in N$ preserves X_Σ
- Orbit of a point with ϕ_α all nonzero is noncompact, so if X contains such a point model is singular
- These are determined by a polynomial $\Delta_{\mathscr{B}}$ Gelfand, Kapranov. Zelevinsky
- GKZ show that this occurs for b in the $(\mathbb{C}^*)^d$ orbits of points satisfying

$$\sum_{\beta\in\mathscr{B}} \langle \beta, \alpha\rangle b_\beta = 0 \quad \forall \alpha \in \mathscr{A} \ , \tag{14}$$

Other Orbits

- Other singularities can be associated to orbits along which some ϕ_α vanish
- These must span a cone τ in $\mathscr{A}$ and if this is not a face the resulting toric subvariety $Z_{\Sigma;\tau}$ is compact
- Υ face of $\operatorname{Conv}\mathscr{A}$ and $\Upsilon^\perp \subset M_\mathbb{R}$ then W restricted to $Z_{\Sigma;\Upsilon}$ contains monomials from $\mathscr{B} \cap \Upsilon^\perp$
- Thus consider faces of $\operatorname{Conv}\mathscr{B}$ in boundary of $\operatorname{Cone}(\operatorname{Conv}\mathscr{A})^\vee$. For such a face noncompact orbits will occur at vanishing of $\Delta_{\mathscr{B}\cap\Gamma}$

The GKZ Determinant

- GKZ study the $\mathscr{B}$-determinant $E_{\mathscr{B}}$ which determines simultaneous vanishing of $\phi_\alpha \partial_\alpha W(\phi)$
- This is given by

$$E_{\mathscr{B}} = \prod_{\Gamma} \Delta_{\mathscr{B}\cap\Gamma}^{u(\Gamma)}, \tag{15}$$

- Product over faces proceeds from dimension zero upwards:
  1. All vertices are included.
  2. Higher dimensional faces are included only when they add points obeying nontrivial affine relations.
- $u(\Gamma)$ positive integers, return to these

Good Families for $\mathscr{B}$

- If $\mathscr{A}$ and $\mathscr{B}$ are *reflexive* faces of Conv $\mathscr{B}$ dual to faces of Conv $\mathscr{A}$ so the product is over simultaneous vanishing of subsets of ϕ_α
- GKZ show: the variety determined by simultaneous vanishing of $\phi_\alpha \partial_\alpha W$ is codimension one and corresponds to the determinant function $E_{\mathscr{B}}$. This determinant is generically nonzero.
- Can further show if all orbits are compact X_Σ is compact
- If $\mathscr{A}$, $\mathscr{B}$ reflexive we find nonsingular models for generic b

Twisted Superpotential

- Classically, free σ fields lead to singularity for ρ in faces of cones of secondary fan
- This prediction is subject to quantum corrections, governed by a twisted superpotential holomorphic in q_a Witten; Morrison, MRP
- At large, generic values for σ chiral fields massive and integrating them out at one-loop leads to

$$\widetilde{W} = \frac{1}{2\pi\sqrt{2}} \sum_{a=1}^{r} \Sigma_a \left[\log q_a - \sum_{\alpha} Q_\alpha^a \log \left(\sum_{b=1}^{r} Q_\alpha^b \Sigma_b \right) \right] \quad (16)$$

Coulomb Branch

- Critical points of $\widetilde{W}$ are

$$q_a = \prod_\alpha \left(\sum_b Q_\alpha^b \sigma_b \right)^{Q_\alpha^a} \tag{17}$$

- Solutions give noncompact component extending to infinity along codimension one locus in $q_a = \prod_\alpha a_\alpha^{Q_\alpha^a}$
- This occurs in $(\mathbb{C}^*)^d$ orbits containing a solution to

$$a_\alpha = \sum_b Q_\alpha^b \sigma_b \tag{18}$$

or equivalently

$$\sum_{\alpha \in \mathscr{A}} \langle \beta, \alpha \rangle a_\alpha = 0 \quad \forall \beta \in \mathscr{B} \tag{19}$$

Mixed Branches

- For a subgroup $\widehat{G}_h \subset \widehat{G}$ let $\widehat{G}_c = \widehat{G}/\widehat{G}_h$
- Define

$$\begin{aligned} \mathscr{A}_h &= \{\alpha \in \mathscr{A} : \pi_h Q e_\alpha = 0\} \\ \mathscr{A}_c &= \mathscr{A} - \mathscr{A}_h. \end{aligned} \tag{20}$$

  partitioning $\mathscr{A}$ into charged and neutral fields under $\mathfrak{g}_c = \mathrm{Hom}(\widehat{G}_c, \mathbb{C})$
- Give σ a large value in $\mathfrak{g}_c$ then $\mathscr{A}_c$ are massive and integrating leads to a product of twisted LG for $\sigma \in \mathfrak{g}_c$ and a reduced GLSM given by $\mathscr{A}_h$ and G_h and suitably restricted superpotential

Mixed Branches

- Find flat noncompact σ directions in twisted LG model when

$$q_a = \prod_{\alpha \in \mathscr{A}_c} \left(\sum_b Q^b_{c,\alpha} \sigma_b \right)^{Q^a_{c,\alpha}} \tag{21}$$

- Reduced GLSM flows to a Coulomb branch, merging with a larger component unless secondary fan for $\mathscr{A}_h \subset N_h$ is complete, or
  1. No point in $\mathscr{A}_h \subset N_h$ should lie at the origin. $\mathscr{A}_c$ spans the subspace $N_{c,\mathbb{R}} \subset N_{\mathbb{R}}$ and so we require $\mathscr{A}_h \cap N_{c,\mathbb{R}} = \emptyset$. That is, $\mathscr{A}_c = N_{c,\mathbb{R}} \cap \mathscr{A}$.
  2. The subspace $N_{c,\mathbb{R}}$ must meet $\operatorname{Conv} \mathscr{A}$ along a *face*.

The $\mathscr{A}$-Determinant and Mirror Symmetry?

- We almost reproduce the GKZ $\mathscr{A}$-determinant, up to u factors, except for vertices of $\operatorname{Conv}\mathscr{A}$
- These represent points in compactification of $\mathscr{M}_A$, possibly associated to decompactification of a component of Z_Σ
- If $\operatorname{Crit} W$ meets these loci we can find singular behavior in the limit
- In $\mathscr{M}_B$ these points generally nonsingular
- We have enough to conclude: If $\mathscr{A}$ and $\mathscr{B}$ are reflexive then the generic element of the family is nonsingular
- Obviously this condition is mirror symmetric

Extremal Transitions

- Extremal transitions will in general connect reflexive to non-reflexive models
- Dropping $\alpha \in \mathscr{A}$ sets $a_\alpha = 0$ picking out a class of triangulations of $\mathscr{A}$
- In general this will typically generate an "exoflop" phase with a growing component sticking out of a singularity in CY component Aspinwall, Greene; Addington,Aspinwall
- Smooth the CY dropping the extra component by a deformation (adding new points to $\mathscr{B}$)
- Check that there are no remaining singularities to blow up by adding points to $\mathscr{A}$
- In general this takes reflexive model to non-reflexive

A Sufficient Condition?

Definition

The pointsets $(\mathscr{A}, \mathscr{B})$ are $\mathscr{B}$-complete if

$$\mathrm{Conv}\big(\mathrm{Cone}(\mathrm{Conv}\,\mathscr{A})^\vee \cap M \cap H_\nu\big) = \mathrm{Conv}(\mathscr{B}), \qquad (22)$$

and similarly $\mathscr{A}$-complete if

$$\mathrm{Conv}\big(\mathrm{Cone}(\mathrm{Conv}\,\mathscr{B})^\vee \cap N \cap H_\mu\big) = \mathrm{Conv}(\mathscr{A}), \qquad (23)$$

- We Conjecture: If a GLSM is defined by pointsets $\mathscr{A}$ and $\mathscr{B}$ which are both $\mathscr{A}$-complete and $\mathscr{B}$-complete then this model is nonsingular for generic values of the parameters
- This is weaker than reflexivity, preserved by transitions, and mirror symmetric
- It is still too strong

$\mathbb{P}^5_{\{2,1,1,1,1,1\}}[3,4]$

$$A^t = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -2 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad B^t = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 2 & -1 \\ 1 & 0 & -1 & -1 & -1 & 4 & 0 \\ 1 & 0 & -1 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & -1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & -1 & 3 & 0 & 0 \\ 1 & 0 & -1 & -1 & -1 & 0 & 4 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 1 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 3 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} \tag{24}$$

$$\mu = (1,1,0,0,0,0,0) \qquad \nu = (1,1,0,0,0,0,0) \tag{25}$$

$$W = \phi_8(\phi_2^3+\phi_3^3+\phi_4^3+\phi_5^3+\phi_6^3+\phi_1\phi_2+\phi_1\phi_3+\phi_1\phi_4+\phi_1\phi_5+\phi_1\phi_6) + \phi_7(\phi_1^2+\phi_2^4+\phi_3^4+\phi_4^4+\phi_5^4+\phi_6^4), \tag{26}$$

Can include 126 internal points in $\mathscr{B}$, get a smooth model in non-Gorenstein ambient space

$\mathbb{P}^5[4,2]$

$$A^t = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B^t = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 0 & -1 \\ 1 & 0 & 3 & -1 & -1 & -1 & 0 \\ 1 & 0 & -1 & -1 & -1 & -1 & 0 \\ 1 & 0 & -1 & 3 & -1 & -1 & 0 \\ 1 & 0 & -1 & -1 & 3 & -1 & 0 \\ 1 & 0 & -1 & -1 & -1 & 3 & 0 \\ 1 & 0 & -1 & -1 & -1 & -1 & 4 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{27}$$

$$\mu = \nu = (1,1,0,0,0,0,0) \tag{28}$$

$\mathscr{A}$ has two triangulations. As promised, one gives X as $\mathbb{P}^5[4,2]$. The other is a "bad hybrid" LG fibration over $\mathbb{P}^1_{2,4}$.

- Drop $(1,0,0,0,0,0,0) \in \mathscr{A}$, restore reflexivity by adding $(-1,2,1,1,1,1,-2)$ to $\mathscr{B}$
- $\mathscr{A}$ now a simplex, so one phase, a LG orbifold with

$$W = \phi_1^4 + \phi_2^4 + \phi_3^4 + \phi_4^4 + \phi_5^4 + \phi_6^4 + b\phi_7^2 + \ldots, \tag{29}$$

- Transition at $b = 0$, this is the new point added
- For $b \neq 0$ ϕ_7 massive, ignoring it we get $\mathbf{2}^6$ Gepner model

$\mathbb{P}^5_{\{2,1,1,1,1,1\}}[4,3]$

- Return to first example, remove $(1,0,0,0,0,0,0) \in \mathscr{A}$, add $(-3,4,3,3,3,-4,-4)$ to $\mathscr{B}$
- Find a smooth LG orbifold with

$$W = \phi_1^4 + \phi_2^4 + \phi_3^4 + \phi_4^4 + \phi_5^4 + b\phi_6^4 + \phi_7^2 + \ldots, \qquad (30)$$

- Transition at $b = 0$
- This is the same as our previous example, so family of LG orbifolds containing $\mathbf{2}^6$ has transitions to non-reflexive first example as well as reflexive example above

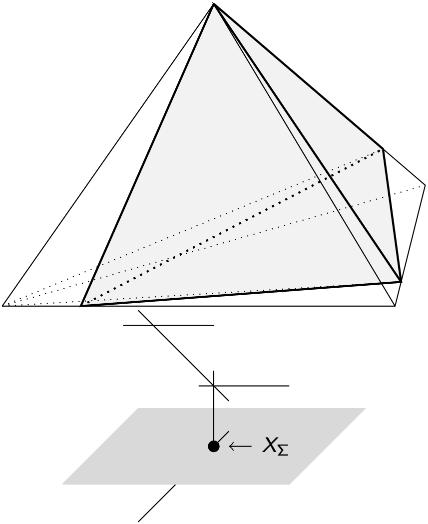
Berglund-Hübsch

- Transposing $P = A^t B$ in LG models gives Berglund-Hübsch mirror symmetry
- Our framework naturally incorporates this. Consider the hypersurface given by

$$x_1^4 x_2 + x_2^4 x_3 + x_3^4 x_4 + x_4^4 x_5 + x_5^5 \qquad (31)$$

- We understand mirrors of quintics. What phase is this for the mirror?
- Using the standard reflexive pair the mirror has too many phases to keep track, so choose $\mathscr{A}$ to be 5 vertices of Conv $\mathscr{A}$ together with 4 more points given by the first 4 monomials
- Find 42 phases. One is just the simplex, mirror to LG model $\mathbf{3}^5/(\mathbb{Z}_5)^4$
- Four triangulations admit the form we started with

Triangulation



Hidden LG Phases

- Z_Σ for these includes as compact component a surface and a set of $\mathbb{P}^1$s
- But Crit W is a point. Locally near this Z_Σ is $\mathbb{C}^5/\mathbb{Z}_{256}$ reproducing the Berglund–Hübsch construction Greene, MRP
- Add $x_1x_2x_3x_4x_5$ with large coefficient, mirror becomes geometric $\mathbb{P}^4_{\{41,48,51,52,64\}}[256]$
- Claim: this is the general pattern
- An alternate version of this is a smooth exception to our conjecture, showing that completeness is too strong a condition

T-Duality and Local MS

- T-duality naturally leads us to consider $W = 0$
- This is a non-compact model, need boundary conditions?
- The dual model naturally has logarithmic D-terms Hori,Vafa
- GKZ determinant gives vanishing of $x_\alpha \partial_\alpha W$
- Following our method to find $\mathscr{A}$-determinant for Z_Σ model recover determinant with multiplicities

Outlook

- We proposed a sufficient condition for a GLSM family to be generically nonsingular. Proof/counterexample?
- Enumerate complete models?
- How likely is a geometric phase? No examples known without one but suspect it is rare? What about large-radius limit more generally?
- Can we state a more refined condition?
- Can we find a precise definition of local model?