New Evidence for $(0, 2)$ Target Space Duality

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$(0,2)$ GLSMs in a nutshell

- Abelian, massive 2D theory $\xrightarrow{\text{IR flow}}$ $(0,2)$ CFT
- $U(1)$ gauge fields $A^{(\alpha)}$, $\alpha = 1, \dots r$
- Chiral superfields: $\{X_i | i = 1, \dots d\}$ charge $(Q_i^{(\alpha)})$, $\{P_l | l = 1, \dots \gamma\}$, charge $(-M_l^{\alpha})$.
- Fermi superfields: $\{\Lambda^a | a = 1 \dots, \delta\}$ charge $N_a^{(\alpha)}$, $\{\Gamma_j^{(\alpha)} | j = 1 \dots c\}$ charge $(-S_j^{(\alpha)})$.
- Gauge and gravitational anomaly cancellation:

$$\sum_{a=1}^{\delta} N_a^{(\alpha)} = \sum_{l=1}^{\gamma} M_l^{(\alpha)} \qquad \sum_{i=1}^{d} Q_i^{(\alpha)} = \sum_{j=1}^{c} S_j^{(\alpha)}$$

$$\sum_{l=1}^{\gamma} M_l^{(\alpha)} M_l^{(\beta)} - \sum_{a=1}^{\delta} N_a^{(\alpha)} N_a^{(\beta)} = \sum_{j=1}^{c} S_j^{(\alpha)} S_j^{(\beta)} - \sum_{i=1}^{d} Q_i^{(\alpha)} Q_i^{(\beta)}$$

for all $\alpha, \beta = 1, ..., r$.

We encapsulate all this information in a table:

X_i				Γ^j			
$Q_1^{(1)}$	$Q_2^{(1)}$	$\dots$	$Q_d^{(1)}$	$-S_1^{(1)}$	$-S_2^{(1)}$	$\dots$	$S_c^{(1)}$
$Q_1^{(2)}$	$Q_2^{(2)}$	$\dots$	$Q_d^{(2)}$	$-S_1^{(2)}$	$-S_2^{(2)}$	$\dots$	$S_c^{(2)}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$Q_1^{(r)}$	$Q_2^{(r)}$	$\dots$	$Q_d^{(r)}$	$-S_1^{(r)}$	$-S_2^{(r)}$	$\dots$	$S_c^{(r)}$

Λ^a				P_l			
$N_1^{(1)}$	$N_2^{(1)}$	$\dots$	$N_\delta^{(1)}$	$-M_1^{(1)}$	$-M_2^{(1)}$	$\dots$	$-M_\gamma^{(1)}$
$N_1^{(2)}$	$N_2^{(2)}$	$\dots$	$N_\delta^{(2)}$	$-M_1^{(2)}$	$-M_2^{(2)}$	$\dots$	$-M_\gamma^{(2)}$
$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$N_1^{(r)}$	$N_2^{(r)}$	$\dots$	$N_\delta^{(r)}$	$-M_1^{(r)}$	$-M_2^{(r)}$	$\dots$	$-M_\gamma^{(r)}$

The GLSM potential

- Superpotential: $S = \int d^2 z d\theta \left[\sum_j \Gamma^j G_j(X_i) + \sum_{l,a} P_l \Lambda^a F_a^l(X_i) \right]$
- G_j and F_a^l are quasi-homogeneous polynomials w/ multi-degrees:

G^j			
S_1	S_2	$\ldots$	S_c

$F_a{}^l$			
$M_1 - N_1$	$M_1 - N_2$	$\ldots$	$M_1 - N_\delta$
$M_2 - N_1$	$M_2 - N_2$	$\ldots$	$M_2 - N_\delta$
$\vdots$	$\vdots$	$\ddots$	$\vdots$
$M_\gamma - N_1$	$M_\gamma - N_2$	$\ldots$	$M_\gamma - N_\delta$

- F-term: $V_F = \sum_j \left| G_j(x_i) \right|^2 + \sum_a \left| \sum_l p_l F_a^l(x_i) \right|^2$
- D-term: $V_D = \sum_{\alpha=1}^r \left(\sum_{i=1}^d Q_i^{(\alpha)} |x_i|^2 - \sum_{l=1}^\gamma M_l^{(\alpha)} |p_l|^2 - \xi^{(\alpha)} \right)^2$
- Transversality condition: $F_a^l(x_i) = 0$ only when $x_i = 0 \ \forall i$
- FI Parameter ($\xi^{(\alpha)} \in \mathbb{R}$) controls the *phases*

- E.g. $\xi > 0 \Rightarrow G_j(X_i) = 0$ and $\langle P \rangle = 0 \Rightarrow$ Geometric phase.
- Geometry: (X, V) with X a CY and bundle described via a monad:

$$0 \to \mathcal{O}_{\mathcal{M}}^{\oplus r_{\mathcal{V}}} \xrightarrow{\otimes E_i^a} \oplus_{a=1}^{\delta} \mathcal{O}_{\mathcal{M}}(N_a) \xrightarrow{\otimes F_a^l} \oplus_{l=1}^{\gamma} \mathcal{O}_{\mathcal{M}}(M_l) \to 0$$

with $V = \frac{ker(F_a^l)}{im(E_i^a)}$

- E.g. $\xi < 0 \Rightarrow \langle p \rangle \neq 0 \Rightarrow$ Non-geometric phase
- E.g. Landau-Ginzburg orbifold w. superpotential:

$$\mathcal{W}(X_i, \Lambda^a, \Gamma^i) = \sum_j \Gamma^j G_j(X_i) + \sum_a \Lambda^a F_a(X_i)$$

- With multiple $U(1)$s, hybrid phases.

Target space duality

- Observation (Distler, Kachru): In LG-phase, G and F on equal footing. Could be interchanged... Algorithm:
- Find phase with one $\langle p_l \rangle \neq 0$ for some l.
- Rescale: $\tilde{\Lambda}^{a_i} := \frac{\Gamma^{j_i}}{\langle p_1 \rangle}$, $\tilde{\Gamma}^{j_i} := \langle p_1 \rangle \Lambda^{a_i}$ $\forall i = 1, \ldots k$ s.t. $\sum_i ||G_{j_i}|| = \sum_i ||F_{a_i}{}^1||$
- Move to a region in bundle moduli space where Λ^{a_i} appear only with P_1 $\forall i \Rightarrow F^l_{a_i} = 0$ $\forall l \neq 1$, $i = 1, \ldots k$.
- Leave non-geometric phase and define new Fermi superfields s.t. $||\tilde{\Lambda}^{a_i}|| = ||\Gamma^{j_i}|| - ||P_1||$ and $||\tilde{\Gamma}^{j_i}|| = ||\Lambda^{a_i}|| + ||P_1||$. item Return to a generic pt. in moduli space to define new TS dual $(0,2)$ GLSM w/ new geometric phase: $(\widetilde{X}, \widetilde{V})$.

Example

x_i							Γ^j		Λ^a				p_l
0	0	0	1	1	1	1	−2	−2	1	0	0	2	−3
1	1	1	2	2	2	0	−4	−5	0	1	1	6	−8

- $SU(3)$ bundle with $dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 68 + 322 = 392$, $h^*(V) = (0, 120, 0, 0)$ (no. of $\mathbf{27}$'s)
- Here $||G_1|| = (2,4)$, $||G_2|| = (2,5)$, $||F_1^1|| = (2,8)$, $||F_2^1|| = (3,7)$, $||F_3^1|| = (3,7)$, $||F_4^1|| = (1,2)$
- Sum of third and fourth F equals sum of two hypersurface degrees.
- Redefine: $\tilde{\Gamma}^1 = \langle p_1 \rangle \Lambda^3$, $\tilde{\Gamma}^2 = \langle p_1 \rangle \Lambda^4$, $\tilde{\Lambda}^3 = \frac{\Gamma^1}{\langle p_1 \rangle}$, $\tilde{\Lambda}^4 = \frac{\Gamma^2}{\langle p_1 \rangle}$, $\tilde{G} = F_3^1$, $\tilde{G}_2 = F_4^1$, $\tilde{F}_3^1 = G_1$, $\tilde{F}_4^1 = G_2$
- Superpotential: $\mathcal{W} = \tilde{\Gamma}^1 \tilde{G}_1 + \tilde{\Gamma}^2 \tilde{G}_2 + \langle p_1 \rangle (\tilde{\Lambda}^3 \tilde{F}_3^1 + \tilde{\Lambda}^4 \tilde{F}_4^1 + \Lambda^1 F_1^1 + \Lambda^2 F_2^1)$

Example

- $||\tilde{G}_1|| = (3,7)$, $||\tilde{G}_2|| = (1,2)$, $||\tilde{F}_3^1|| = (2,4)$, $||F_4^1|| = (2,5)$, $||\tilde{\Gamma}^1|| = (-3,-7)$, $||\tilde{\Gamma}^2|| = (-1,-2)$ $||\tilde{\Lambda}^3|| = (1,4)$, $||\tilde{\Lambda}^4|| = (1,3)$.
- Leads to new geometry $(\widetilde{X}, \widetilde{V})$

x_i						Γ^j	Λ^a				p_l
0	0	0	1	1	1	-3	1	0	1	1	-3
1	1	1	2	2	0	-7	0	1	4	3	-8

- $dim(\widetilde{\mathcal{M}_0}) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V})) = 2 + 95 + 295 = 392$, $h^*(\widetilde{V}) = (0, 120, 0, 0)$
- Here $h^{1,1}$ stays fixed, complex structure and bundle moduli interchange.
- More general mixing possible...

Increasing no. of $U(1)$'s

Can find an alternate description of X to open up more possibilities:

- Add a new coord y_1 with multi-degree B and a new hypersurface also of degree B
- Perform previous procedure (e.g. $||B|| = ||F^1_1|| + ||F^1_2|| - 1$)
- Resolve singularities (Distler, Greene, Morrison) by formally adding a $\mathbb{P}^1$ (another coord y_2)
- Set constraint $G^B = y_1 = 0$ to eliminate y_1. Use additional $U(1)$ and D-term to fix y_2 to a real constant.$\leftrightarrow X\times$ a single pt.
- Leads to $(\widetilde{X}, \widetilde{V})$ with higher $h^{1,1}$
- In general, all numbers of moduli mixed.

x_1	..	x_d	y_1	y_2	Γ^1	..	Γ^c	Γ^B	Λ^1	Λ^1	..	Λ^δ	p_1	p_2	..	p_γ
0	..	0	1	1	0	..	0	-1	0	0	..	0	-1	0	..	0
Q_1	..	Q_d	B	0	$-S_1$	..	$-S_c$	$-B$	N_1	N_2	..	N_δ	$-M_1$	$-M_2$	..	$-M_\gamma$

End up with new geometry:

x_1	..	x_d	y_1	y_2	$\tilde{\Gamma}^1$	..	Γ^c	$\tilde{\Gamma}^B$	$\tilde{\Lambda}^1$	$\tilde{\Lambda}^2$	..	p_1	p_2	..
0	..	0	1	1	-1	..	0	-1	1	0	..	-1	0	..
Q_1	..	Q_d	B	0	$-(M_1 - N_1)$	..	$-S_c$	$-(M_1 - N_2)$	0	$M_2 - B$	..	$-M_1$	$-M_2$	..

- Can choose B (e.g. $B = 0$) to make this a conifold transition between $X \leftrightarrow \widetilde{X}$ ("Transgression", Candelas, et al).
- Can repeat this many times. In general all moduli mixed (and any one can be held fixed).
- What to make of this TS duality?
- Two possibilities
  1. Two distinct theories, connected in moduli space (e.g. like conifold transitions in Type II theories)
  2. A true duality (i.e. isomorphism) of target space theories

The question...

- In 2011, Blumenhagen + Rahn performed a landscape scan. Tested duality by counting states:

$$h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(End_0(\widetilde{V}))$$

  and charged matter in $\sim 80,000$ examples. Agreement in nearly all cases.
- Question: Can we test this duality (even in the geometric, perturbative regime) in more detail?
- Recall, these are $N = 1$ 4D theories. Want more than $dim(\mathcal{M}_0)$... $\Rightarrow$ Moduli can be obstructed!
- Can we compare the effective potential and vacuum space of the chain of dual theories?
- Must engineer examples with interesting/calculable potentials...

Our starting point...

- Thanks to recent progress (LA, Gray, Lukas, Ovrut) know more about how to do this...
- Conditions for $N = 1$ Supersymmetry in $4D$: Hermitian-Yang Mills Eqns $F_{ab} = F_{\bar{a}\bar{b}} = g^{a\bar{b}} F_{\bar{b}a} = 0$
- $g^{a\bar{b}} F_{\bar{b}a} = 0 \Leftrightarrow$ Donaldson-Uhlenbeck-Yau Thm: V is slope, poly-stable.
- $F_{ab} = F_{\bar{a}\bar{b}} = 0$: V is holomorphic.
- Stability $\Leftrightarrow$ $4D$ D-terms
- Holomorphy $\Leftrightarrow$ $4D$ F-terms
- Can we test TS duality for examples with non-trivial moduli obstructions?
- Will choose simple e.g.s: Ordinary CICYs, $0 \to V \to B \to C \to 0$, $c_2(TX) = c_2(V)$

Stability

- The slope, $\mu(V)$, of a vector bundle is

$$\mu(V) \equiv \frac{1}{\mathrm{rk}(V)} \int_X c_1(V) \wedge \omega \wedge \omega$$

  where $\omega = t^k \omega_k$ is the Kahler form on X (ω_k a basis for $H^{1,1}(X)$).

- V is Stable if for every sub-sheaf, $\mathcal{F} \subset V$, with $0 < rk(\mathcal{F}) < rk(V)$,

$$\mu(\mathcal{F}) < \mu(V)$$

- V is Poly-stable if $V = \bigoplus_i V_i$, V_i stable such that $\mu(V) = \mu(V_i)\ \forall i$

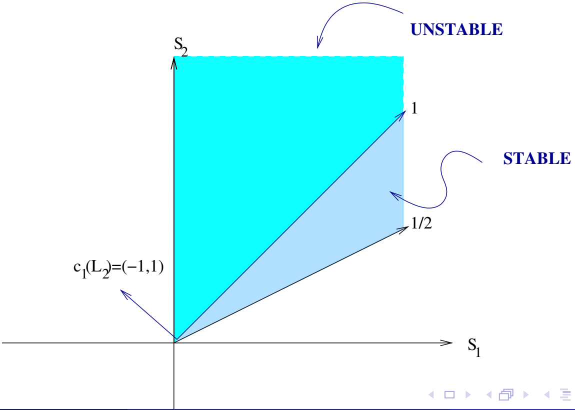
- Conservation of Misery $\rightarrow$ Tough to find sub-sheaves.

- Monad E.g. CY 3-fold, $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^3 & 4 \end{array} \right]$, with $h^{1,1} = 2$.
- V an $SU(3)$ bundle defined by

  $0 \to V \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(1,-1) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \xrightarrow{f} \mathcal{O}_X(2,1) \to 0$

  which is destabilized in part of the Kähler cone by the rank 2 sub-bundle

  $0 \to \mathcal{F} \to \mathcal{O}_X(1,0) \oplus \mathcal{O}_X(0,1)^{\oplus 2} \to \mathcal{O}_X(2,1) \to 0$ with $c_1(\mathcal{F}) = -\omega_1 + \omega_2$.



Poly-stable locus

- On a line (in general a hyperplane) in Kähler moduli space, the sub-sheaf $\mathcal{F}$ becomes important
- Can describe V in terms of this sub-sheaf as $0 \to \mathcal{F} \to V \to V/\mathcal{F} \to 0$
- Space of such extensions given by $Ext^1((V/\mathcal{F}), \mathcal{F}) = H^1(X, \mathcal{F} \otimes (V/\mathcal{F})^\vee)$, where the origin of this group is a locus in the moduli space of V for which $V = \mathcal{F} \oplus V/\mathcal{F}$, with $c_1(\mathcal{F}) = -c_1(V/\mathcal{F})$
- On the line with $\mu(\mathcal{F}) = 0$, for SUSY to exist, need $V = \bigoplus_i V_i = \mathcal{F} \oplus V/\mathcal{F}$ to have a poly-stable bundle.
- This means the structure group changes!
  $SU(3) \to S[U(2) \times U(1)]$. Locally $S[U(2) \times U(1)] \approx SU(2) \times U(1)$
- Enhancement of symmetry $\to E_6 \times U(1)$. New $U(1)$ gauge field in the visible $4d$ theory. (Sharpe)

- E.g. $SU(3) \rightarrow S[U(2) \times U(1)]$.
- The enhanced $U(1)$ is Green-Schwarz massive.
- Matter fields and "moduli" are now charged under this U(1).

  Locally, $E_8 \supset E_6 \times SU(2) \times U(1)$

  $248 \rightarrow (1,1)_0 + (1,2)_{-3/2} + (1,2)_{3/2} + (1,3)_0 + (78,1)_0 + (27,1)_1 + (27,2)_{-1/2} + (\bar{27},1)_{-1} + (\bar{27},2)_{1/2}$
- Bundle moduli decompose as

$$H^1(V \otimes V^\vee) \rightarrow \begin{cases} H^1(\mathcal{F} \otimes \mathcal{F}^\vee) + H^1(\mathcal{F} \otimes \mathcal{K}^\vee) + H^1(\mathcal{K} \otimes \mathcal{F}^\vee) \\ (1,3)_0 \quad + \quad (1,2)_{-3/2} \quad + \quad (1,2)_{3/2} \end{cases}$$

- E_6 Matter: $H^1(V) \rightarrow \begin{cases} H^1(\mathcal{K}) + H^1(\mathcal{F}) \\ (27,1)_1 + (27,2)_{-1/2} \end{cases}$

- The complexified Kähler moduli, $T^k = t^k + 2i\chi^k$, transform with a shift symmetry through the axion, χ^k
- $U(1)$ D-term contribution to the $4d$ effective potential (Sharpe, Lukas, Stelle, Blumenhagen, Weigand, Honecker,…).

$$D^{U(1)} \sim \frac{\mu(\mathcal{F})}{Vol(X)} - \sum_{M,\bar{N}} Q^M G_{M\bar{N}} C^M \bar{C}^{\bar{N}}$$

with (FI)-term $\sim \mu(\mathcal{F})$ -the slope of the relevant sub-bundle $\mathcal{F}$, C^M are $U(1)$ charged fields.

- This D-term potential is independent of complex structure moduli for all anomaly free and $N = 1$ SUSY theories.
- Stability walls can lead to transitions between bundles (S-equivalence classes, etc).
- Kähler cone substructure can lead to constraints on phenomenology: Yukawa textures, etc.

Holomorphic Vector bundles

- V holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle and then vary the complex structure? Must a bundle stay holomorphic for any variation $\delta\mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No
- $0 \to V \otimes V^\vee \to \mathcal{Q} \stackrel{\pi}{\to} TX \to 0$ is known as the Atiyah sequence.
- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \stackrel{d\pi}{\to} H^1(TX) \stackrel{\alpha}{\to} H^2(V \otimes V^\vee) \to \dots$$

- If the map $d\pi$ is surjective then $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus H^1(TX)$
- But $d\pi$ not surjective in general! $H^1(\mathcal{Q}) = H^1(V \otimes V^\vee) \oplus Im(d\pi)$
- $d\pi$ difficult to define, but by exactness, $Im(d\pi) = Ker(\alpha)$ where $\alpha = [F^{1,1}] \in H^1(V \otimes V^\vee \otimes TX^\vee)$ is the Atiyah Class

Deformation Theory

There are three objects in deformation theory that we need

- $Def(X)$: Deformations of X as a complex manifold. Infinitesimal defs parameterized by the vector space $H^1(TX) = H^{2,1}(X)$. These are the *complex structure* deformations of X.
- $Def(V)$: The deformation space of V (changes in connection, δA) *for fixed* C.S. moduli. Infinitesimal defs measured by $H^1(End(V)) = H^1(V \otimes V^\vee)$. These define the *bundle moduli* of V.
- $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$ where

$$0 \to V \otimes V^\vee \to \mathcal{Q} \stackrel{\pi}{\to} TX \to 0$$

  If $\mathcal{P}$ is the total space of the bundle, $\mathcal{Q} = r_* T\mathcal{P}$.
- $H^1(X, \mathcal{Q})$ are the actual complex moduli of a heterotic theory

GVW Superpotential and F-terms

- For the $4d$ Theory: We have Gukov-Vafa-Witten superpotential

  $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}} \left(\omega^{3\mathrm{YM}} - \omega^{3\mathrm{L}} \right)$

- In Minkowski vacuum (with $W = 0$), F-terms:

  $F_{C_i} = \frac{\partial W}{\partial C_i} = -\frac{3\alpha'}{\sqrt{2}} \int_X \Omega \wedge \frac{\partial \omega^{3\mathrm{YM}}}{\partial C_i}$

- Dimensional Reduction Anzatz: $A_\mu = A_\mu^{(0)} + \delta A_\mu + \bar{\omega}^i_\mu \delta C_i + \omega^i_\mu \delta \bar{C}_i$

$$F_{C_i} = \int_X \epsilon^{\bar{a}\bar{c}\bar{b}} \epsilon^{abc} \Omega^{(0)}_{abc} 2\bar{\omega}^{xi}_{\bar{c}} \,\mathrm{tr}(T_x T_y) \left(\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]} \right)$$

- Computationally: $Ker(\alpha)$: Free C.S. moduli. $Im(\alpha)$: lifted C.S. moduli.
- Superpotential observations in lit. since 80's. Hard part is engineering calculable examples.
- Idea: Build bundles where "ingredients" crucially depend on complex structure...

Our present program...

- Build monad bundles with F-term and D-term obstructions
- Analyze the vacuum space
- Systematically construct all TS dual geometries
- Compare...
- Note: Counting is easiest... so begin w/ examples that lift, rather than just constrain moduli
- Caveat: Many interesting questions to ask of the GLSM, for this talk I will focus solely on comparing the target space theories generated in the geometric phases.

D-term Example

x_i						Γ^j	Λ^a								p_l		
1	1	0	0	0	0	−2	1	−1	0	0	2	1	1	2	−3	−1	−2
0	0	1	1	1	1	−4	−1	1	1	1	1	2	2	2	−2	−4	−3

with initial total moduli count

$$dim(\mathcal{M}_0) = h^{1,1}(X) + h^{2,1}(X) + h^1(X, End_0(V)) = 2 + 86 + 340 = 428$$

What is the effective target space theory in the geometric phase?

- Naively: $rk(V) = 5$, $c_1(V) = 0 \Rightarrow SU(5)$ 4D theory.
- Not so fast though...Mixed positive/negative entries in Λ^a indicate sub-sheaves.
- In fact, V stable *only* on a ray in Kähler moduli space $t^2 = 4t^1$.
- Only supersymmetric configuration of V: $V \to U_3 \oplus L \oplus L^\vee$ w/ $L = \mathcal{O}(1, -1)$.

- Non-Abelian Enhancement: Structure group is not $SU(5)$, rather $S[U(1) \times U(1)] \times SU(3) \subset E_8 \Rightarrow SU(6) \times U(1)$, with $U(1)$ symmetry Green-Schwarz massive.

Field	Cohom.	Multiplicity	Field	Cohom.	Multiplicity
$\mathbf{1}_{+2}$	$H^1(L \otimes L)$	0	$\mathbf{1}_{-2}$	$H^1(L^\vee \otimes L^\vee)$	10
$\mathbf{15}_0$	$H^1({U_3}^\vee)$	0	$\overline{\mathbf{15}}_0$	$H^1(U_3)$	80
$\mathbf{20}_{+1}$	$H^1(L)$	0	$\mathbf{20}_{-1}$	$H^1(L^\vee)$	0
$\mathbf{6}_{+1}$	$H^1(L \otimes U_3)$	72	$\mathbf{6}_{-1}$	$H^1(L^\vee \otimes U_3)$	90
$\overline{\mathbf{6}}_{+1}$	$H^1(L \otimes {U_3}^\vee)$	0	$\overline{\mathbf{6}}_{-1}$	$H^1(L^\vee \otimes {U_3}^\vee)$	2
$\mathbf{1}_0$	$H^1(U_3 \otimes {U_3}^\vee)$	166			

Table : *Particle content of the $SU(6) \times U(1)$ theory associated to the bundle along its reducible and poly-stable locus $V = \mathcal{O}(-1,1) \oplus \mathcal{O}(1,-1) \oplus U_3$ (i.e. on the stability wall).*

Features of interest

- Non-trivial D-term lifts one Kähler modulus. Reduction in moduli $dim(\mathcal{M}_1) = dim(\mathcal{M}_0) - 1 = 427$
- Bundle forced to locus w/ non-Abelian symmetry enhancement
- From the stability wall, can explore branch structure into nearby geometries.
- How much of this is visible in the TS duals?
- In this case can construct a chain of 17 TS dual geometries w/ conifold-type "splits" $\rightarrow h^{1,1} + 1$. What do we get?...

TS dual of D-term e.g.

One example:

x_i								Γ^j		Λ^a								p_l		
0	0	0	0	0	0	1	1	−1	−1	0	0	1	0	0	0	0	0	0	0	−1
1	1	0	0	0	0	0	0	−2	0	1	−1	0	0	2	1	1	2	−3	−1	−2
0	0	1	1	1	1	0	0	−2	−2	−1	1	−1	1	3	2	2	2	−2	−4	−3

Questions:

1. Does $(\widetilde{X}, \widetilde{V})$ give rise to a stability wall?
2. What is $dim(\widetilde{\mathcal{M}}_0)$? Is $dim(\widetilde{\mathcal{M}}_1)$ reduced by the same amount?
3. Does the structure group of $\widetilde{V}$ reduce as well, leading to a 4-dimensional non-Abelian enhancement of symmetry?
4. Do the charged matter spectra of the two theories match?
5. Does the vacuum branch structure (i.e. local deformation space) correspond?

TS Dual

- First pass yields a discrepancy...

  $dim(\tilde{\mathcal{M}}_0) = h^{1,1}(\widetilde{X}) + h^{2,1}(\widetilde{X}) + h^1(X, End_0(\widetilde{V})) = 3 + 55 + 371 = 429$

- However, this isn't quite the right count. Three de-stabilizing sub-sheaves for $\widetilde{V}$. Stability condition on the wall reduces free Kähler moduli by 2 (two D-terms). $\Rightarrow dim(\widetilde{\mathcal{M}_1}) = 427$. Agreement!
- Once again, $\widetilde{V}$ is *forced* to be reducible: $\tilde{V} \to (\tilde{L}_1 \oplus \tilde{L}_1^\vee) \oplus (\tilde{L}_2 \oplus \tilde{U}_2)$
- Structure group is $S[U(1) \times U(1)] \times S[U(1) \times U(2)] \Rightarrow$ $SU(6) \times U(1) \times U(1)$ w/ both $U(1)$s GS massive.
- First three questions manifestly answered in the positive!

Field	Cohom.	Multiplicity	Field	Cohom.	Multiplicity
$\mathbf{1}_{+2,0}$	$H^1(\tilde{L}_1 \otimes \tilde{L}_1)$	0	$\mathbf{1}_{-2,0}$	$H^1(\tilde{L}_1^\vee \otimes \tilde{L}_1^\vee)$	10
$\mathbf{15}_{0,-1}$	$H^1(\tilde{U}_2^\vee)$	0	$\overline{\mathbf{15}}_{0,+1}$	$H^1(\tilde{U}_2)$	80
$\mathbf{15}_{0,+2}$	$H^1(\tilde{L}_2^\vee)$	0	$\overline{\mathbf{15}}_{0,-2}$	$H^1(\tilde{L}_2)$	0
$\mathbf{20}_{+1,0}$	$H^1(\tilde{L}_1)$	0	$\mathbf{20}_{-1,0}$	$H^1(\tilde{L}_1^\vee)$	0
$\mathbf{6}_{+1,-2}$	$H^1(\tilde{L}_1 \otimes \tilde{L}_2)$	0	$\mathbf{6}_{-1,-2}$	$H^1(\tilde{L}_1^\vee \otimes L_2)$	0
$\mathbf{6}_{+1,+1}$	$H^1(\tilde{L}_1 \otimes \tilde{U}_2)$	72	$\mathbf{6}_{-1,+1}$	$H^1(\tilde{L}_1^\vee \otimes \tilde{U}_2)$	90
$\overline{\mathbf{6}}_{+1,-1}$	$H^1(\tilde{L}_1 \otimes \tilde{U}_2^\vee)$	0	$\overline{\mathbf{6}}_{-1,-1}$	$H^1(\tilde{L}_1^\vee \otimes \tilde{U}_2^\vee)$	0
$\overline{\mathbf{6}}_{+1,+2}$	$H^1(\tilde{L}_1 \otimes \tilde{L}_2^\vee)$	0	$\overline{\mathbf{6}}_{-1,+2}$	$H^1(\tilde{L}_1^\vee \otimes \tilde{L}_2^\vee)$	2
$\mathbf{1}_{0,0}$	$H^1(\tilde{U}_2 \otimes \tilde{U}_2^\vee)$	99	$\mathbf{1}_{0,-3}$	$H^1(\tilde{L}_2 \otimes \tilde{U}_2^\vee)$	0
$\mathbf{1}_{0,+3}$	$H^1(\tilde{L}_2^\vee \otimes \tilde{U}_2)$	98			

Table : *Particle content of the* $SU(6) \times U(1) \times U(1)$ *theory* $\leftrightarrow$ $V = \tilde{L}_1 \oplus \tilde{L}_1^\vee \oplus \tilde{L}_2 \oplus \tilde{U}_2$.

Branch Structure

- Various branches. E.g. Search for breaking: $SU(6) \to SU(5)$ stable off the wall. $(L + L^\vee + U_3 \to V_5)$

$$D^{U(1)}_{GS} \sim \frac{3}{16}\frac{\epsilon_S {\epsilon_R}^2 \mu(\mathcal{L})}{{\kappa_4}^2 \mathcal{V}} - \frac{1}{2}\left((-2)|C_{-2,0}|^2 + (+1)|C_{+1,-5}|^2 + (-1)|C_{-1,-5}|^2 + (-1)|C_{-1,+5}|^2\right)$$

$$D^{U(1)}_{SU(6)} \sim \frac{1}{2}\left((-5)|C_{+1,-5}|^2 + (-5)|C_{-1,-5}|^2 + (+5)|C_{-1,+5}|^2\right)$$

- D-flat solns require $\mu(L) < 0 \Rightarrow$ One-sided stability chamber
- Can show that this branch is described via

x_i						Γ^j	Λ^a									p_l			
1	1	0	0	0	0	−2	1	0	0	0	0	2	1	1	2	−1	−3	−1	−2
0	0	1	1	1	1	−4	−1	1	1	1	1	1	2	2	2	−1	−2	−4	−3

(1)

- Shares the reducible (wall) locus $L + L^\vee + U_3$ with

$$0 \to L \to \mathcal{O}(0,1)^{\oplus 2} \to \mathcal{O}(1,1) \to 0$$

- $dim(\mathcal{M}_0) = dim(\mathcal{M}_1) = h^{1,1}(X) + h^{2,1}(X) + h^1(End_0(V)) = 2 + 86 + 338 = 426$

TS Dual and branches

- Again, search for a branch $SU(6) \to SU(5)$

$$D^{U(1)}_{GS1} \sim \frac{3}{16}\frac{\epsilon_S {\epsilon_R}^2 \mu(\mathcal{L}_1)}{{\kappa_4}^2 \mathcal{V}} - \frac{1}{2}\left((-2)|C_{-2,0,0}|^2 + (+1)|C_{+1,+1,-5}|^2 + (-1)|C_{-1,+1,-5}|^2 + (-1)|C_{-1,+2,+5}|^2\right)$$

$$D^{U(1)}_{GS2} \sim \frac{3}{16}\frac{\epsilon_S {\epsilon_R}^2 \mu(\mathcal{L}_2)}{{\kappa_4}^2 \mathcal{V}} - \frac{1}{2}\left((+3)|C_{0,+3,0}|^2 + (+1)|C_{+1,+1,-5}|^2 + (+1)|C_{-1,+1,-5}|^2 + (+2)|C_{-1,+2,+5}|^2\right)$$

$$D^{U(1)}_{SU(6)} \sim \frac{1}{2}\left((-5)|C_{+1,+1,-5}|^2 + (-5)|C_{-1,+1,-5}|^2 + (+5)|C_{-1,+2,+5}|^2\right)$$

- Again, one such branch w/ $\mu(L_2) > 0$ and $\mu(L_1) < 0$

- Described via the new monad

x_i								Γ^j		Λ^a									p_l		
0	0	0	0	0	0	1	1	−1	−1	0	0	0	1	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	−2	0	1	0	0	0	0	2	1	1	2	−1	−3	−1
0	0	1	1	1	1	0	0	−2	−2	−1	1	1	−1	1	3	2	2	2	−1	−2	−4

Again $dim(\widetilde{\mathcal{M}}) = 426$ and $SU(5)$ charged matter match exactly.

Branch Structure

- Even more exciting, we now have many examples where we can describe all branches via monads. In general, find commutative diagram:

$$\begin{array}{ccccccc} & V_1 & \overset{dual}{\longrightarrow} & \widetilde{V}_1 & \\ \langle C \rangle & \downarrow & & \downarrow & \langle \tilde{C} \rangle \\ & V_2 & \overset{dual}{\longrightarrow} & \widetilde{V}_2 & \end{array}$$

- Only intriguing exceptions: In some cases stability wall in TS dual is located on the boundary of Kähler moduli space (i.e. some $t^i = 0$, but $Vol(X) >> 0$). Here effective theories difficult to compare. Perturbative/non-perturbative duality?...

F-term Example

- What about e.g.s w/ holomorphy obstructions? (i.e. F-term lifting)? For this we have to work a little harder...
- Consider the $SU(2)$ bundle

x_i						Γ^j	Λ^a			p_l
1	1	0	0	0	0	-2	2	-1	-1	0
0	0	1	1	1	1	-4	0	2	2	-4

- Does not define a stable bundle for general choices of complex structure: Missing a map $F_1^a \in H^0(X, \mathcal{O}(-2,4)) = 0$ generically. However, line bundle cohomology can jump...
- Shown in (arXiv:1107.5076) that on a 53-dim. sublocus of CS moduli space, $h^0(X, \mathcal{O}(-2,4)) = 1$. $\Rightarrow dim(\mathcal{M}_1) = dim(\mathcal{M}_0) - 33$.
- But... also proved that when $X \leftrightarrow \widetilde{X}$ connected via geometric transitions, dim(jumping locus)= $dim(\mathcal{CS}_{jump} \cap \mathcal{CS}_{shared})$

Target Space Dual theory of F-term example

x_i								Γ^j		Λ^a			p_l
0	0	0	0	0	0	1	1	−1	−1	0	1	0	−1
1	1	0	0	0	0	0	0	−1	−1	2	−2	0	0
0	0	1	1	1	1	0	0	−2	−2	0	0	4	−4

- Fortunately, new bundle $\widetilde{V}$ now involves two jumping map components.
- $h^0(\widetilde{X}, \mathcal{O}(0,-2,4)) = 1$ fixes 15 CS moduli
- $h^0(\widetilde{X}, \mathcal{O}(1,-2,4)) = 1$ fixes 18 CS moduli
- Stability walls form chamber structure but do not lift moduli.
- In total $dim(\widetilde{\mathcal{M}}_1) = dim(\widetilde{\mathcal{M}}_0) - 33$ as required!
- Charged matter spectra also agree.
- Fine print: easy e.g. but $c_2(V) \neq c_2(TX)$, need to add a spectator

x_i						Γ^j	Λ^a				p_l	
1	1	0	0	0	0	−2	0	0	2	2	−2	−2
0	0	1	1	1	1	−4	1	1	1	1	−2	−2

Conclusions and Future work

- In many non-trivial cases, we have found that not just the matter spectrum, but the effective potentials and vacuum spaces of the target space dual theories agree.
- We have found new evidence that target space duality may give hints towards a true $(0, 2)$ string duality...
- Can we establish a deeper isomorphism between sigma models?
- Many open questions to explore in the GLSMs...
- Intriguing to carry this analysis further, calculate Yukawa couplings, etc.
- Links to other string dualities?